MATH 396 PROBLEMS 14

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Regular problems:

1. Let V be a $\mathbb{C}$-vector space with inner product denoted by $< u, v >$. Then V is in particular an $\mathbb{R}$-vector space (by using the $\mathbb{R}$-scalar multiplication). Decide if the following functions are real inner products on V:

(a) $Re < u, v > + Im < u, v >$

(b) $Re < u, v >$

(c) $Im < u, v >$.

[Here $Re(x + iy) = x$, $Im(x + iy) = y$ for $x, y \in \mathbb{R}$.]

2. Calculate $< \sin(mx), \sin(nx) >$, $< \sin(mx), \cos(nx) >$ in $L^2([0, 2\pi], \mathbb{R})$.

3. For constants K, r, s, determine numbers a, b and a function $f : [a, b] \to \mathbb{R}$, $f(a) = r$, $f(b) = s$, such that the solid formed by rotating the graph of f around the x axis in $\mathbb{R}^3$ has surface K and maximal possible volume. [Use the same method as in Problem 5 from last time: parametrize $(x, f(x)) = (\phi(x), \psi(x))$ such that the derivative of the rotational surface area is 1: $\psi \sqrt{(\phi')^2 + (\psi')^2} = 1$. The derivative of the rotational volume is $\psi^2 \phi'$ (up to constants). Substitute for ϕ' from the previous equality, and form a first order Euler equation for ψ.]

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Challenge problem:

4. In the center of a circular pond is a beaver. An animal control worker on shore runs K times faster than the beaver swims. He is trying to catch the beaver without swimming in the pond. For what values of K can the beaver escape the pond without being caught, and for what values will he be caught? What is the optimal strategy for the beaver to escape?