MATH 396 PROBLEMS 2

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Regular problems:

1. Calculate the derivative (by t) of the function
$$F(t) = \int_1^2 \frac{e^{tx}}{x} dx.$$
[Do not try to integrate directly.]

2. Calculate:
$$\int_{[0,1]\times[0,1]} \frac{dxdy}{\sqrt[3]{xy}}.$$

3. Using Fubini's theorem, calculate
$$\int_{[0,1]\times[0,1]} \frac{dxdy}{x+y}.$$

4. Using polar coordinates, calculate
$$\int_B \frac{xy}{x^2+y^2} dxdy$$
where B is the solid circle of radius 1 with center in the origin.

Challenge problem:

5. (a) Define 3-dimensional coordinates of a point (x, y, z) as (r, α, β) where (r, α) are the 2-dimensional polar coordinates of (x, y) and β is the angle of the vector (x, y, z) with the plane determined by the x and y axes. Find the Jacobian of the map expressing the 3-dimensional Cartesian coordinates x, y, z in terms of the polar coordinates r, α, β.

(b) Using (a), calculate:

$$\int_B \frac{dxdydz}{(\sqrt{x^2 + y^2 + z^2})^t}$$

where B is the set of all points of distance ≤ 1 from 0, and t is a parameter.

(c) Can you generalize (a) to dimension n?

(d) For which values of t does

$$\int_B \frac{dx_1 ... dx_n}{(\sqrt{x_1^2 + ... + x_n^2})^t}$$

converge where B is the ball in $\mathbb{R}^n$ with radius 1 and center in the origin? [half credit for a correct conjecture...]