MATH 396 PROBLEMS 5

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Regular problems:

1. (a) Find a way to represent the unit circle $\{(x,y)|x^2+y^2=1\}$ as a piecewise smooth curve.

(b) Calculate the curve integral of the first kind (=line integral with respect to length) of the function $f(x,y)=|xy|$ over the unit circle.

2. Find a piecewise smooth curve whose set of points is the intersection of the two surfaces $\{(x,y,z)|x^2+y^2=z^2\}$ and $\{(x,y,z)|y^2=z\}$.

3. Calculate the curve integral of the first kind (=line integral with respect to length) of the function $f(x,y)=x$ over the segment of a parabola $\{(x,x^2)|0\leq x\leq 1\}$.

4. Calculate the curve integral of the first kind (=line integral with respect to length) of the function $f(x,y)=x+y$ over the triangle in the plane with vertices $(0,0)$, $(1,0)$ and $(0,1)$.

Challenge problems:

5. Prove in (more) detail that there exists a Peano curve, i.e. a continuous function $f : [0, 1] \to \mathbb{R}^2$ completely filling a square. [Construct - or show the existence of - a sequence of functions uniformly converging to the curve; use compactness.]

6. Two pieces of rope (wick), when lit on one side, take exactly 1 hour each to burn across. However, the speed of burning is not constant, and the ropes are not identical. Can you find a way to measure exactly 45 minutes using the ropes?