MATH 396 PROBLEMS 6

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Regular problems:

1. Calculate the line integral (=curve integral of second kind) of the function
$f\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x^2 - 2xy \\ y^2 - 2xy \end{pmatrix}$ from $\begin{pmatrix} -1 \\ 1 \end{pmatrix}$ to $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ along the parabola $y = x^2$.

2. A force field (i.e function assigning to each point a force vector) in $\mathbb{R}^3$ is given by the formula
$$f\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} yz \\ xz \\ x(y+1) \end{pmatrix}.$$
Calculate the work done by f in moving a particle once around the triangle with vertices $\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ 1 \\ -1 \end{pmatrix}$ in that order.

3. Calculate the complex curve integral of the function $f(z) = 1/z$ counterclockwise over
(a) the circle in $\mathbb{C}$ with center in the origin and radius r
(b) the square with vertices in the points $1 + i, -1 + i, -1 - i, 1 - i$
(c) the circle with radius 1 and center 2.

4. Using Greene's formula, evaluate the line integral (=curve integral of second kind) of the function $f\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} xe^{-y^2} \\ -x^2ye^{-y^2} + 1/(x^2 + y^2) \end{pmatrix}$ around the boundary of the square consisting of all points $\begin{pmatrix} x \\ y \end{pmatrix}$ such that $|x| \leq 1, |y| \leq 1$.

Challenge problem:

5. Calculate the complex curve integral of the function z^n, $n \in \mathbb{Z}$ over the unit circle in $\mathbb{C}$.