MATH 396 PROBLEMS 8

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Regular problems:

1. Compute the area of a sphere of radius a using surface integrals.

2. If S is the surface of the sphere $x^2 = y^2 + z^2 = a^2$, compute the surface integral of second kind

$$\int_S (xz, yz, x^2).$$

3. The cylinder $x^2 + y^2 = 2x$ cuts out a portion of a surface S from the upper nappe of the cone $x^2 + y^2 = z^2$. Compute the value of the surface integral of the first kind

$$\int_S (x^4 - y^4 + y^2z^2 - z^2x^2 + 1).$$

4. Let S be the hemisphere $x^2 + y^2 + z^2 = 1$, $z \geq 0$. Using Stokes' theorem, calculate

$$\int_S (y^2, xy, xz).$$

Challenge problems:

5. Let A be a $k \times n$ matrix, $k \leq n$. Prove that the sum of the squares of the determinants of all $k \times k$ submatrices of A is equal to $det(AA^T)$.

6. Can you find a function $f : \mathbb{R} \to \mathbb{R}$ such that $f(f(x)) = e^x$ for all x?