MATH 396 PROBLEMS 9

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Regular problems:

1. Let S be the image of $[0,1]\times[0,1]$ under the map $\mathbb{R}^2 \to \mathbb{R}^3$ given by $x = r+s^2$, $y = r-s$. Using Stokes' theorem, calculate the integral of $f(x,y,z) = (x^2+y, z-y, y^3)$ over the boundary of S.

2. Prove that if $f : \mathbb{C} \to \mathbb{C}$ satisfies $f(z) \in \mathbb{R}$ for all $z \in \mathbb{C}$, and f has complex derivative everywhere, then f is constant. [Consider the total differential.]

3. Using Gauss' formula, calculate the surface integral of second kind of the function $f(x,y,z) = (xy, xz, z^2)$ over the surface of the unit ball $\{(x,y,z) | x^2+y^2+z^2 \leq 1\}$ (oriented so that the normal vector points outward).

4. (a) Using the definition, calculate the complex derivative of the function $f(z) = \frac{1}{z-i}$.

(b) A function $f : \mathbb{C} \to \mathbb{C}$ is given by $f(x+iy) = e^{x^2+y^2+1}(\cos(2xy) + i\sin(2xy))$. Is it holomorphic? [Use Cauchy-Riemann conditions.]

Challenge problem:

5. Consider the 2-form in $\mathbb{R}^4$

$$\omega = (x + t + z)^2 dx \wedge dy + e^x dy \wedge dt + (x - y) dx \wedge dt + 3xy dx \wedge dz.$$

(a) Let S be the image of the square $[0, 1] \times [0, 1]$ under the map $\mathbb{R}^2 \to \mathbb{R}^4$ given by $x = rs$, $y = r^2 + s^2$, $z = r - s$, $t = s$. Calculate $\int_S \omega$.

(b) Using Stokes' theorem, calculate the integral of ω over the boundary of the cube $\{(x, y, z, t) | 0 \leq x, y, z \leq 1, t = 2\}$.