Math 425 9/7/2011

Note Title 9/7/2011

If we have n equally likely possible outcomes, then the probability of k of them occurring is

$$\frac{k}{n}.$$

The basic principle of counting:

If we have two independent experiment the first of which has m possible outcomes and the second of which has n possible outcomes, then the number of possible outcomes of both experiments together is $\underline{m} \cdot \underline{n}$.

<u>Proof</u>: this is the number of pairs of outcomes (i, j), $i = 1, \dots, m$
$j = 1, \dots, n$.

This is $m \cdot n$. $\square$

<u>Generalization</u>: If we have $\underline{r}$ experiments which are independent and the numbers of possible outcomes are $n_1, \dots, n_r$ then

the total number of possible outcomes is

$$n_1 \cdot \ldots \cdot n_r.$$

<u>Example</u>: How many possible functions are there on n points if the value of the function at each point is allowed to be 0 or 1?

<u>Solution</u>: 2 possible values at each point
= independent experiments,
so the number of outcomes is

$$\underbrace{2 \cdot 2 \cdot \ldots \cdot 2}_{n \text{ times}} = 2^n. \quad \square$$

Permutations

are arrangement of $\underline{n}$ different objects in $\underline{n}$ places

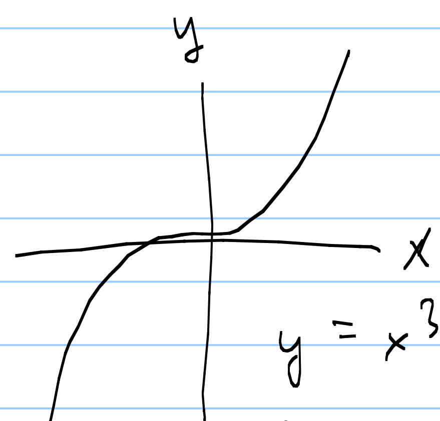
$$\underbrace{\text{maps}}_{\text{functions}} \quad \underbrace{\{1, \ldots, n\}}_{\text{domain}} \longrightarrow \underbrace{\{1, \ldots, n\}}_{\text{codomain}}$$

a rule assigning to each element of the

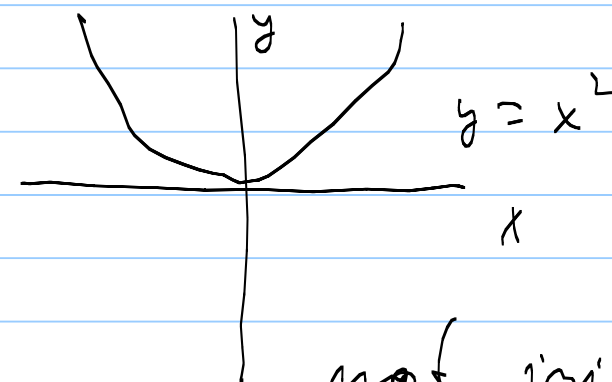
domain precisely one element of the codomain

$\boxed{\text{Permutations are bijective maps.}}$

injective 1 to 1 : every element of the codomain appears as a value at most once

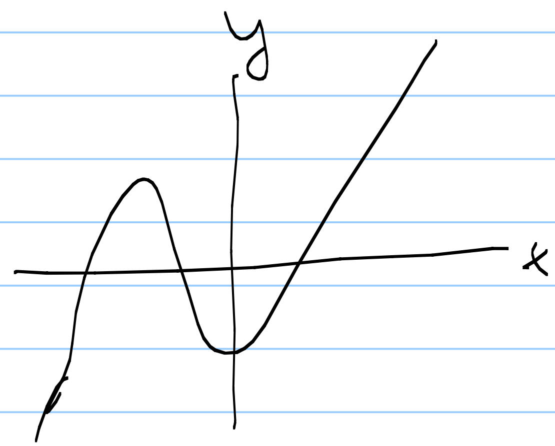


injective

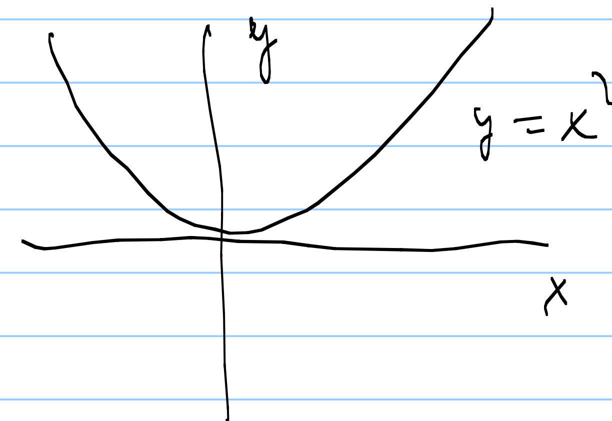


not injective

surjective : every element of the codomain does appear as a value at least once



surjective
(not injective)



not surjective

A bijective map is one which is both surjective and injective

A permutation is a bijective map

$$\{1, \ldots, n\} \longrightarrow \{1, \ldots, n\}$$

<u>Example</u> : $n = 3$ <u>Permutations</u>

$$\begin{array}{ccc}
1 \longmapsto 1 & 1 \longmapsto 1 & 1 \longmapsto 2 \\
2 \longmapsto 2 & 2 \longmapsto 3 & 2 \longmapsto 1 \\
3 \longmapsto 3 & 3 \longmapsto 2 & 3 \longmapsto 3
\end{array}$$

$$\begin{array}{ccc}
1 \longmapsto 2 & 1 \longmapsto 3 & 1 \longmapsto 3 \\
2 \longmapsto 3 & 2 \longmapsto 1 & 2 \longmapsto 2 \\
3 \longmapsto 1 & 3 \longmapsto 2 & 3 \longmapsto 1
\end{array}$$

6 permutations on the set of 3 elements $\{1, 2, 3\}$. Why?

<u>Where can 1 go?</u> $\left.\begin{array}{l}\mapsto 1\\ \mapsto 2\\ \mapsto 3\end{array}\right\}$ 3 choices

After choosing where 1 went,

<u>Where can 2 go?</u> – anywhere <u>except</u> where 1 went } 2 choices

<u>Where can 3 go?</u> – anywhere <u>except</u> where 1 and 2 went } 1 choice

Total number of choices: $3 \cdot 2 \cdot 1 = 6$.

Similarly, the number of permutations on the $\underline{n}$ element set $\{1, \dots, n\}$ is

$$n \cdot (n-1) \cdot (n-2) \cdot \dots \cdot 1 = \underbrace{n!}_{\text{"n factorial"}}.$$

Example (p. 4, Example 3c modified)

Ms. Jones has: 4 math books
3 chem. books

2 history books

How many ways can she arrange them on a shelf so that the books of each subject are next to each other?

<u>Solution</u>: $\underbrace{}_{\text{math}}\ \underbrace{}_{\text{chem}}\ \underbrace{}_{\text{history}}\ \underbrace{}_{\boxed{\text{order of subjects}}}$

$$4! \cdot 3! \cdot 2! \cdot 3! = 24 \cdot 6 \cdot 2 \cdot 3 = \ldots$$

$\square$

<u>Example 3d</u>: How many different words (real or made up) can be made from

the word PEPPER using every letter precisely once?

Solution: Permutations on 6 elements: $6!$.

But certain arrangement count as the same word:

$$\underbrace{3!}_{\text{arrangement of P's}} \cdot \underbrace{2!}_{\text{arrangements of the E's}} \cdot \underbrace{1!}_{\text{arrangement of the R's}} = 12$$

Correct answer to the problem: $\dfrac{6!}{3!\cdot 2!\cdot 1!} = 60.$ $\square$

In general: If we have n objects such that n_1 of them are the same, n_2 are the same but different from the first n_1, ..., n_k are the same but different from the remaining ones,

$$n = n_1 + \dots + n_k$$

then the total number of different arrangements is

$$\frac{n!}{n_1! \cdot \dots \cdot n_k!}.$$

$\boxed{\text{HW}}$: ① Which of the following functions with domain and codomain $\mathbb{R}$ (= real numbers) are injective, surjective and/or bijective?

$$f : \mathbb{R} \longrightarrow \mathbb{R}$$

ⓐ $f(x) = x^2 + 2x$

ⓑ $f(x) = x^4 + x + 1$

ⓒ $f(x) = x^3 - 2x^2 + 1$

ⓓ $f(x) = x^5$.

② How many different words (real or

made up) can be made from the letters of the word MISSISSIPPI where each letter must be used exactly once?