Math 425 9/12/2011

Note Title 9/12/2011

Axioms of Probability

Sample space = the set of all possible outcomes of an experiment } S

Examples ① Determining the gender of a newborn child:

$S = \{ \text{girl}, \text{boy} \}$

② A horse race among 7 horses

$$S = \{\text{all the permutations on } \{1, \dots, 7\}\} \quad 7!$$

③ Flipping two coins

ordered

$$S = \{(H,H), (H,T), (T,H), (T,T)\}$$

set

$$4 = 2 \cdot 2$$

(4) Measuring in hours the lifetime of a light-bulb:

$$S = \{x \in \mathbb{R} \mid x \geq 0\} = [0, \infty)$$

An <u>event</u> E is a subset of S.

Can all subsets be events? Not necessarily.

(let's not worry about it yet)

Examples: In example (2), we could put

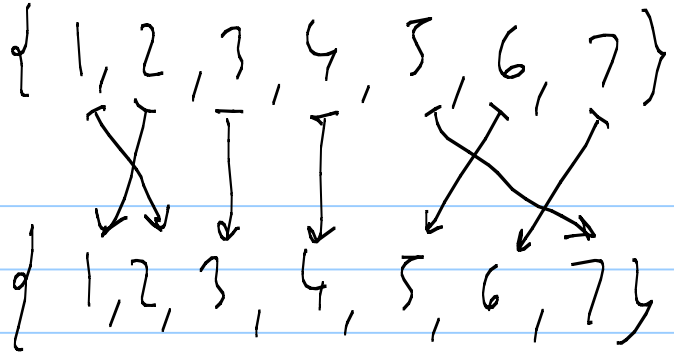
$E = \{$ Flock 3 wins $\} =$

$= \{$ all the permutations of $\{1, \dots, 7\}$ where the first element is 3 $\}$

Comment: Usually, we represent permutations by sequences, e.g.

$$(2, 1, 3, 4, 6, 7, 5) \notin E$$

corresponds to the map



which book ends up on what place?

(Note: sometimes one reverses the arrows in the correspondence between permutations and bijections.)

In Example (4):

E = {the light bulb lasts less than 5 hours}

$$E = \{x \in [0,\infty) \mid 0 \leq x < 5\} = [0,5)$$

$$(S = [0,\infty))$$

Conclusion: Probability has a lot to do with sets. Basic operations with set.

$E \cup F$ = all the elements which are either in E, or in F, or in both

$$= \{x \mid x \in E \text{ or } x \in F\}$$

UNION OF TWO SETS

Example ② (Horse race).

E = Horse 3 wins

Suppose

F = Horse 2 wins

$E \cup F$ = Horse 2 wins or Horse 3 wins

$= \{$ permutation on $\{1, \ldots, 7\}$ | 2 or 3 are first (when representing a permutation as a sequence) $\}$

Intersection of two sets

$$E \cap F = EF = \{ x \mid x \in E \text{ and } x \in F \}$$

↗ book notation

In the above example where E = Horse 3 wins,

F = Horse 2 wins,

$$E \cap F = \emptyset$$ ↖ the EMPTY SET

Let G = Horse 7 placed 2nd

$E \cap G$ = Horse 3 won and horse 7 placed 2nd

$= \{(3,7,\dots) \in \text{permutations on } \{1,\dots,7\}\}$.

Quiz on Wed

The complement of a set E **in S**.

The complementary event:

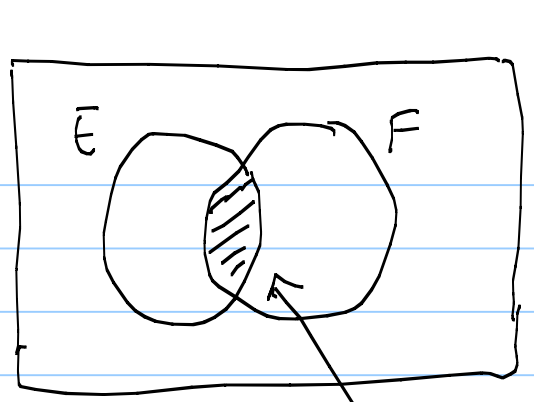
$$E^c = \{x \in S \mid x \notin E\}.$$

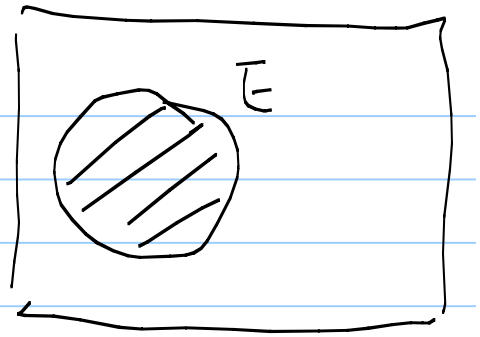
In the above example (the horse race)

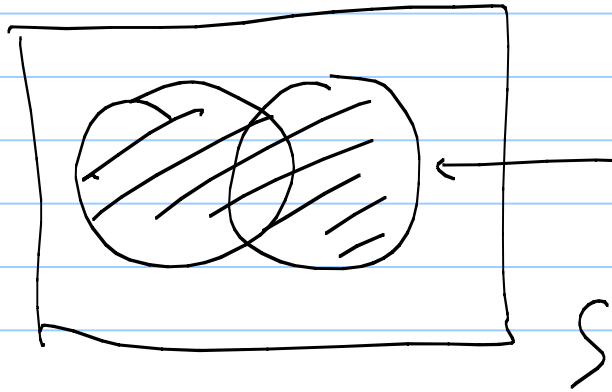
E = {Mark} wins

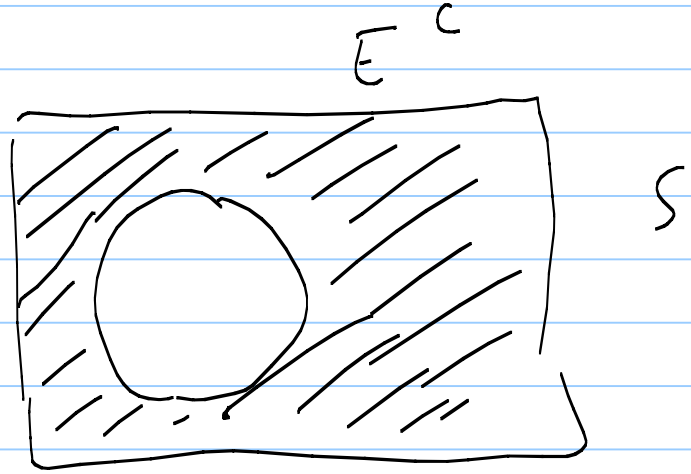
E^c = {Horse} does not win.

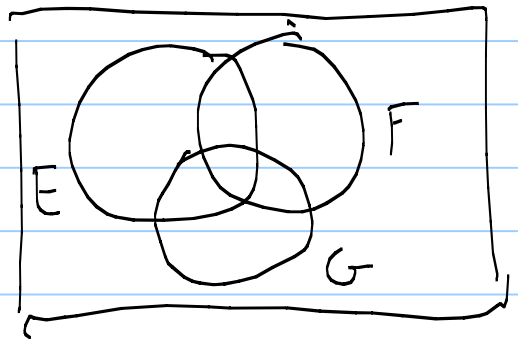
Venn diagrams











An event E is contained in an event H

$$E \subseteq H$$

(book uses $\subset$ allowed) ($E = H$ is)

when every element of E is contained in H.

Example: $E \subseteq E \cup F$.

HW: (1) In the example of a race

Between 7 horses, let K be the event:

$K = \{$ Horse 1 wins and horse number 2 does not place second. $\}$

What are the number of outcomes in K?

(2) Theoretical exercise 2. on p. 54.

All HW due Wed. (Note: Quiz).