MATH 425 9/14/2011

Note Title 9/14/2011

Answer to the quiz: Think of no vote as "none of the above". (ordered) Partitions of n into $\underbrace{k+1}_{l}$ non-negative integers.

$$\binom{n+l-1}{l-1}$$

$$\boxed{\binom{n+k}{k}}$$

De Morgan rules:

(1) $$\left(\bigcup_{i=1}^{n} E_i\right)^c = \bigcap_{i=1}^{n} (E_i)^c$$

(2) $$\left(\bigcap_{i=1}^{n} E_i\right)^c = \bigcup_{i=1}^{n} (E_i)^c$$

Proof of (1): $x \in \bigcup_{i=1}^{n} E_i$ <u>means</u>: x is an element of at least one of the sets E_i, $i=1,\dots,n$.

$x \in \left(\bigcup_{i=1}^{n} E_i\right)^c$ <u>means</u>: x is not an element of any of the sets E_i, $i=1,\dots,n$

That is the same thing as $x \notin E_1$, $x \notin E_2$, ..., $x \notin E_n$.

That is the same thing as $x \in (E_1)^c$, $x \in (E_2)^c$, ..., $x \in (E_n)^c$.

That is the same thing as $x \in \bigcap_{i=1}^{n} (E_i)^c$. $\square$

This also means, if $E_i = (F_i)^c$,

$$\left(\bigcup_{i=1}^{n} (F_i)^c\right)^c = \bigcap_{i=1}^{n} \left((F_i)^c\right)^c = \bigcap_{i=1}^{n} F_i$$

Take the complement of this identity:

$$\bigcup_{i=1}^{n} (F_i)^c = \left(\bigcap_{i=1}^{n} F_i\right)^c$$

This is the same as (2).

Axioms of Probability

The naive approach: If I repeat the exact same experiment n times, assume the experiment has a random outcome and the event E occurs $n(E)$ times, then

$$P(E) = \lim_{n \to \infty} \frac{n(E)}{n}.$$

① We don't actually know if randomness exists in the real universe.

(2) If randomness actually exists, we don't know what it is.

The approach we adopt is that we <u>assume</u> probability is assigned to events

$P(E)$ E

and we try to study its properties.

If the sample space S is <u>infinite</u> then not every event E may have a probability.

An event that has a probability is called <u>measurable</u>.

<u>Axiom 1</u>: $0 \leq P(E) \leq 1$

<u>Axiom 2</u>: S is measurable and $P(S) = 1$

<u>Axiom 3</u>: If E_i are measurable <u>disjoint</u> events

$i = 1, 2, \ldots$ $\qquad i \neq j \Rightarrow E_i \cap E_j = \emptyset$

then $\bigcup_{i=1}^{\infty} E_i$ is measurable and

$$P\left(\bigcup_{i=1}^{\infty} E_i\right) = \sum_{i=1}^{\infty} P(E_i).$$

Axiom 4: If E is measurable, so is E^c.

NO HW ASSIGNED TODAY