425 9/21/2011

Note Title 9/21/2011

① 5a When we roll two dice, what is the probability that the total on the two dice will be 7?

<u>Solution</u>: All possibilities

$$|S| = \#(S) = 6 \cdot 6 = 36$$

sample space

The event we are counting contains outcomes:

$1+6$, $2+5$, $3+4$, $4+3$, $5+2$, $6+1$ } 6 outcomes

All outcomes are equally likely, so

$$P(E) = \frac{6}{36} = \frac{1}{6}. \quad \square$$

(2) In the previous experiment, what is the probability that the total will be 12?

Solution: $\left.6+6\right\}$ 1 outcome

Answer: $\frac{1}{36}$.

(3) Example 56 A bowl contain 6 white and 5 black balls. We draw 3 balls. What

is the probability that one is white and the other two are black?

Solution 1: We can count ordered selections.

$|S| = \#(S) = 11 \cdot 10 \cdot 9$

$\boxed{\begin{array}{l} 6 \text{ white} \\ 5 \text{ black} \end{array}}$

$|E| = |E_1| + |E_2| + |E_3|$

E_1: first ball white; E_2: second ball white; E_3: third ball white — other balls black

$6 \cdot 5 \cdot 4$ $\quad$ $5 \cdot 6 \cdot 4$ $\quad$ $5 \cdot 4 \cdot 6$

$$P(E) = \cancel{3} \cdot \frac{\overset{2}{\cancel{6}} \cdot \cancel{5} \cdot 4}{11 \cdot \cancel{10} \cdot \cancel{9}} = \frac{4}{11}.$$

<u>Solution 2</u>: Counting unordered selections. (combinations)

$$|S| = \binom{11}{3}$$

" subset of 3 element in an 11-element set

$$|E| = \binom{6}{1} \cdot \binom{5}{2}$$

$$P(E) = \frac{\binom{6}{1} \cdot \binom{5}{2}}{\binom{11}{3}} = \frac{\cancel{6} \cdot \cancel{10} \cdot \overset{2 \cdot \cancel{3}}{2 \cdot \cancel{3}}}{11 \cdot \cancel{10} \cdot \cancel{9}} = \frac{4}{11}. \quad \square$$

4 Example 3c : n balls in a bowl, 1 wins.
If I draw k balls, what is my chance of winning?

Solution: Use unordered selections.

$$|S| = \binom{n}{k}.$$

non-winning

$$|E| = \binom{n-1}{k-1}$$

$$\frac{|E|}{|S|} = \frac{\binom{n-1}{k-1}}{\binom{n}{k}} = \frac{\frac{(n-1)\cdot\ldots\cdot(n-k+1)}{1\cdot\ldots\cdot(k-1)}}{\frac{n\cdot(n-1)\cdot\ldots\cdot(n-k+1)}{1\cdot\ldots\cdot(k-1)\cdot k}} = \frac{k}{n}.$$

$\square$

HW: Suppose we have n balls and 1 is winning. I have k draws, but after each draw, I throw the ball back. What is my chance of winning? (drawing the winning ball at least once).

5 Example 5f: A poker hand consists of 5 cards. If the cards have consecutive values and are <u>not all</u> of the same suit, we call it a <u>straight</u>. What is the probability of being dealt a straight?

<u>Solution</u>: All possible different hands:

$$|S| = \binom{52}{5}.$$

(Count Ace as 1)

A 2 3 4 5 6 7 8 9 10 J Q K A

1 2 . . . . 13 14

(1, 2, 3, 4, 5)

.

.

(10, J, Q, K, A)

10 possible sets of values.

Each card is of one of the four suits ♣ ♢ ♡ ♠

$4^5 - 4$

↖ all the cards are of the same suit

$$|E| = (4^5 - 4) \cdot 10 \qquad P(E) = \frac{(4^5-4)\cdot 10}{\binom{52}{2}} \doteq 0.0039$$

$\square$

(6) A full house : 3 cards of the same value

5g + 2 cards of the same value.

What is the probability of a full house?

Solution : $|S| = \binom{52}{5}$

If we choose the values : 3 Aces, 2 Jacks : $\binom{4}{3} \cdot \binom{4}{2}$

The number of pairs of values : $13 \cdot 12$

$|E| = 13 \cdot 12 \cdot \binom{4}{3} \cdot \binom{4}{2}$

$P(E) = \dfrac{13 \cdot 12 \cdot \binom{4}{3} \cdot \binom{4}{2}}{\binom{52}{5}} \doteq 0.0014$

HW:

1. Drawing k balls out of n balls one of which is winning, and throwing the ball back after each draw, what is the likelihood of drawing the winning ball at least once?

2. 16 on p. 51

3. 28 on p. 52