

425

9/21/2011

Note Title

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① 5a When we roll two dice,
what is the probability that the total
on the two dice will be 7?

Solution: All possibilities

$$|S| = \#(S) = 6 \cdot 6 = 36$$

↑ sample space

The event we are counting contains outcomes:
 $1+6, 2+5, 3+4, 4+3, 5+2, 6+1 \}$ 6 outcome

All outcomes are equally likely, so

$$P(E) = \frac{6}{36} = \frac{1}{6}. \quad \square$$

② In the previous experiment, what is the probability that the total will be 12?

Solution : $6+6$ } 1 outcome

Answer : $\frac{1}{36}$,

③ Example 5) A bowl contain 6 white
and 5 black balls. We draw 3 balls. What

is the probability that one is white and the other two are black?

Solution: We can count ordered selections.

$$|S| = \#(S) = 11 \cdot 10 \cdot 9$$

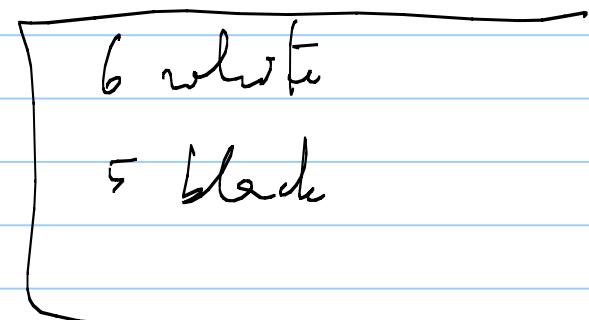
$$|E| = |E_1| + |E_2| + |E_3|$$

↑ ↑ ↑
first ball second ball third ball
white white white

$$6 \cdot 5 \cdot 4$$

$$5 \cdot 6 \cdot 4$$

$$5 \cdot 4 \cdot 6$$



} other balls
black

$$P(E) = 3 \cdot \frac{6 \cdot 5 \cdot 4}{11 \cdot 10 \cdot 9} = \frac{4}{11}.$$

Solution 2: Counting unordered selections.
(combinations)

$$|S| = \binom{11}{3}$$

"subset of 3 element in an 11-element set"

$$|E| = \binom{6}{1} \cdot \binom{5}{2}$$

$$P(E) = \frac{\binom{6}{1} \cdot \binom{5}{2}}{\binom{11}{3}} = \frac{\cancel{6} \cdot \cancel{10} \cdot \cancel{2} \cdot 3}{\cancel{11} \cdot \cancel{10} \cdot \cancel{9}} = \frac{4}{11}. \quad \square$$

④

Example 5c : n balls in a bowl, 1 wins,

If I draw k balls, what is my chance of winning?

Solution : Use unordered selections.

$$|S| = \binom{n}{k}.$$

$$|E| = \binom{n-1}{k-1}$$


non-winning

$$\frac{|E|}{|S|} = \frac{\binom{m-1}{k-1}}{\binom{m}{k}} = \frac{(m-1) \cdot \dots \cdot (m-k+1)}{1 \cdot \dots \cdot (k-1) \cdot \cancel{(m)} \cdot (m-1) \cdot \dots \cdot (m-k+1)} = \frac{k}{m}.$$

$1 \cdot \dots \cdot (k-1) \cdot \cancel{k}$ □

(Hw): Suppose we have m balls and 1 is winning. I have k draws, but after each draw, I throw the ball back. What is my chance of winning? (drawing the winning ball at least once).

⑤ Example 5f: A poker hand consist of 5 cards.

If the cards have consecutive values and are not all of the same suit, we call it a straight. What is the probability of being dealt a straight?

Solution: All possible different hands:

$$|S| = \binom{52}{5}.$$

(Count Ace as 1)

A 2 3 4 5 6 7 8 9 10 J Q K A

1 2

13 14

(1, 2, 3, 4, 5)

(10, J, Q, K, A)

10 possible sets of values.

Each card is of one of the four suits ♠ ♦ ♣ ♤

$$4^5 - 4$$

↑ all the cards are of the same suit

$$|E| = (4^5 - 4) \cdot 10$$

$$P(E) = \frac{(4^5 - 4) \cdot 10}{\binom{52}{2}} = 0.0039$$

□

⑥ A full house : 3 cards of the same value
+ 2 cards of the same value.

What is the probability of a full house?

Solution : $|S| = \binom{52}{5}$

If we choose the values:

$$\begin{array}{c} 3 \text{ Aces} \\ 2 \text{ Jacks} \end{array} : \binom{4}{3} \cdot \binom{4}{2}$$

The number of pairs of values: $13 \cdot 12$

$$|E| = 13 \cdot 12 \cdot \binom{4}{3} \cdot \binom{4}{2} \quad \left| P(E) = \frac{13 \cdot 12 \cdot \binom{4}{3} \cdot \binom{4}{2}}{\binom{52}{5}} = 0.0014 \right.$$

(Hw):

① Drawing k balls out of n balls one of which is winning, and throwing the ball back after each draw, what is the likelihood of drawing the winning ball at least once?

② 16 on p. 51

③ 28 on p. 52