

MATH 425

9/26/2011

Note Title

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Quiz Wednesday

Conditional Probability

Suppose we have two events E, F where $P(E) > 0$. Define the conditional probability of F assuming E

as

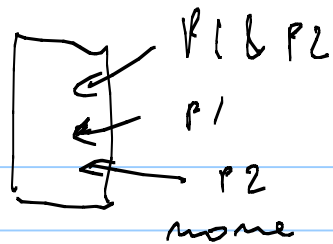
$$P(F|E) = \frac{P(E \cap F)}{P(E)}.$$

Example: There is a game show where there are two doors. There are two prizes randomly placed behind the doors. The player gets to open one door "for practice". If there is at least one prize behind that door, the player wins one prize.

Then the doors are closed again and the prize that appeared during the practice round (if any) is removed. The player is then asked to open again one of the doors. If there is a prize behind it, the player wins it. Q: What is the right strategy?

OPEN SAME DOOR / OPEN THE OTHER DOOR / IT DOES NOT MATTER

Solution:



What is the probability of the event E that the player sees a prize during the practice round?

$$P(E) = \frac{3}{4}$$

F_1 : Both prizes are behind the door the player opened during the practice round.

$$P(F_1) = \frac{1}{4}$$

The probability of winning by choosing the same door again is

$$P(F_1|E) = \frac{1/4}{3/4} = \frac{1}{3}.$$

F_2 : There is one prize behind each door.

$$P(F_2) = \frac{1}{2}$$

The probability of winning by switching doors is

$$P(F_2|E) = \frac{1/2}{3/4} = \frac{2}{3}.$$

Answer: Choose the other door!

Example 2: If I have two children, and ^(at least) one of my children is a boy, what is the probability that I have two boys?

Solution: S: Child 1 Child 2

(B	,	B)
(B	,	G)
(G	,	B)
(G	,	G)

E = At least one of my children is a boy.

$$P(E) = \frac{3}{4}$$

F = Both of my children are boys

$$P(F) = \frac{1}{4}$$

Answer: $P(F|E) = \frac{1/4}{3/4} = \frac{1}{3}$.

Example 3: Suppose I meet a kid (boy) in a cafe and he mentions that he has exactly one sibling. What is the probability his sibling is a boy?

Solution: The probability is $\frac{1}{2}$.

In a more complicated way,

		Child 1	Child 2	I met	T
$ S = 8$	$\rightarrow$	B	B	1	$\leftarrow$
		B	G	1	$\leftarrow$
		G	B	1	
		G	G	1	
	$\rightarrow$	B	B	2	$\leftarrow$
		B	G	2	
		G	B	2	$\leftarrow$
		G	G	2	

$$E = I \text{ met a boy.} \quad P(E) = \frac{4}{8} = \frac{1}{2}$$

$$F = \text{Both are boys}$$

$$P(F) = \frac{2}{8} = \frac{1}{4}$$

$$P(F|E) = \frac{1/4}{1/2} = \frac{1}{2}.$$

Comment: I always chose $F \subseteq E$ in my solutions, so $E \cap F = F$.

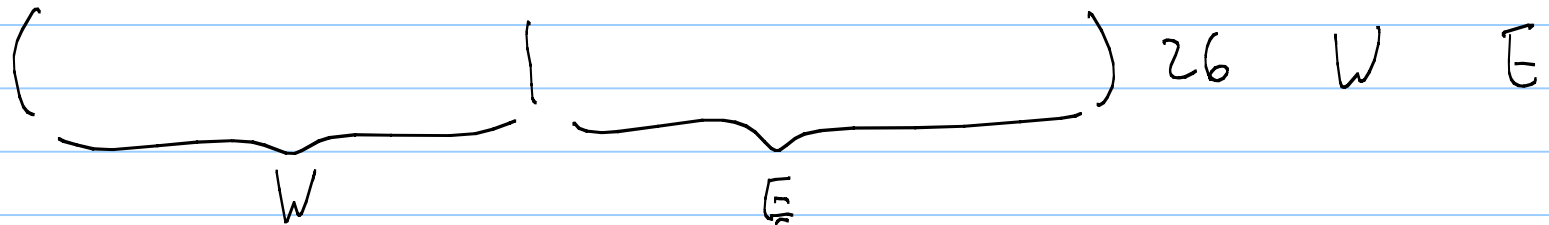
Example: 2c on p. 60

In bridge (52 cards, 4 players each get 13 cards)

Suppose N and S have ^{precisely} a total of 8 spades

among them. What is the probability E has ^{exactly} 3 of the remaining 5 spades?

Solution: Reduced sample space:



$$|S| = \binom{26}{5}$$

$F = E$ has precisely 3 spades

$$|F| = \binom{13}{3} \binom{13}{2}$$

$$\frac{\binom{13}{3} \binom{13}{2}}{\binom{26}{5}}$$

$=$

Work:

$$\frac{\binom{5}{3} \binom{21}{10}}{\binom{26}{13}}$$

S = The 13 cards
(out of 26) F got

F = 3 spades out of 5
any other cards out of
 $26 - 5$

Example: de p. 61

Probability of getting A in French ... $\frac{1}{2}$

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in Chemistry ... $\frac{2}{3}$.

If I flip a coin to decide which class to take, what is the probability of getting an A in chemistry?

Solution: E = choosing chemistry

F = getting an A.

$$P(E) = \frac{1}{2}$$

$$\frac{2}{3} = P(F|E)$$

$$\frac{1}{2} = P(F|E^c)$$

$$\frac{P(E \cap F)}{P(E)} = P(F|E).$$

$$P(E \cap F) = P(F|E) \cdot P(E) = \frac{2}{3} \cdot \frac{1}{2} = \frac{1}{3}.$$

Quiz on Wednesday!

HW (1) Suppose one of my children is a boy born on a Wednesday. Assuming I have exactly two children, what is the probability they are both boys?

② 3.31 on p. 104