MATH 425 9/28/2011

Note Title 9/28/2011

Solution to the quiz:

A = $\underbrace{\text{at least}}_{\text{out of two}}$ one die comes up a 6

A_1 = die nr. 1 comes up a 6

A_2 = die nr. 2 comes up a 6

$$P(A) = P(A_1) + P(A_2) - P(A_1 \cap A_2)$$

$$= \frac{1}{6} + \frac{1}{6} - \frac{1}{36} = \frac{11}{36}$$

$$P(A_1 \cap A_2) = \frac{1}{36}$$

$$P(A_1 \cap A_2 \mid A) = \frac{1/36}{11/36} = \frac{1}{11} \quad \blacksquare$$

The multiplication rule

$$P(E_1 \cap \cdots \cap E_n) = P(E_n \mid E_1 \cap \cdots \cap E_{n-1}) \cdot P(E_{n-1} \mid E_1 \cap \cdots \cap E_{n-2}) \cdot \cdots \cdot P(E_2 \mid E_1) \cdot P(E_1)$$

Proof: The right hand side is

$$\frac{P(E_n \cap E_1 \cap \cdots \cap E_{n-1})}{\cancel{P(E_1 \cap \cdots \cap E_{n-1})}} \cdot \frac{\cancel{P(E_{n-1} \cap E_1 \cap \cdots \cap E_{n-2})}}{\cancel{P(E_1 \cap \cdots \cap E_{n-2})}} \cdots \frac{\cancel{P(E_2 \cap E_1)}}{\cancel{P(E_1)}} \cdot \cancel{P(E_1)}$$

We have chain cancelation, leaving

$$P(E_1 \cap \cdots \cap E_n). \qquad \square$$

The Bayes Formula

$$P(E) = P(E|F) \cdot P(F) + P(E|F^c) \cdot (1 - P(F)).$$

Proof: The right hand side is:

$$\frac{P(E \cap F)}{\cancel{P(F)}} \cdot \cancel{P(F)} + \frac{P(E \cap F^c)}{\cancel{P(F^c)}} \cdot \cancel{(1 - P(F))} =$$

$$= P(E \cap F) + P(E \cap F^c) = P(E). \qquad \square$$

<u>Example</u> (of Bayes' formula): On a multiple choice test, suppose a question has m answers. Suppose the probability a student knows the answer is p. Assuming a student not knowing the answer chooses the right answer with probability $1/m$, what is the conditional probability the student knew the answer provided they answered correctly.

<u>Solution</u>: C = correct answer

K = knowing the answer

$$P(K \mid C) = \frac{P(K \cap C)}{P(C)} = \frac{P(C \mid K) \cdot P(K)}{P(C \mid K)\, P(K) + P(C \mid K^c) \cdot P(K^c)}$$

$$P(C \mid K) = 1$$

$$P(C \mid K^c) = \frac{1}{m}$$

$$P(K) = p$$

$$= \frac{p}{p + \frac{1}{m}(1-p)}$$

$$= \frac{mp}{mp + (1-p)}$$

$$= \frac{mp}{1 + (m-1)p}$$

HW: 3.27 on p. 103

3.47 on p. 105