MATH 425 9/30/2011

Note Title 9/30/2011

$P(B|A)$ = $\underbrace{\text{probability that the event } B \text{ occurs assuming the event } A \text{ occurs.}}_{\text{conditional probability}}$

$$= \frac{P(A \cap B)}{P(A)}$$

$$P(B|A) \cdot P(A) = P(A \cap B) = P(A|B) \cdot P(B)$$

$$\boxed{P(B|A)\cdot P(A) = P(A|B)\cdot P(B)}$$

Example 3n p.77 A bin of flashlights:

$P($ type 1 lasts >100 hours $) = 0.7$

$P($ type 2 last >100 hours $) = 0.4$

$P($ type 3 last >100 hours $) = 0.3$

There are 20% of type 1, 30% type 2, 50% type 3.

(b) Suppose a flashlight lasts over 100 hours.
What are the probabilities it is of type 1, 2 or 3?

Solution:

T_i = of type T_i

A = lasted over 100 hours

$P(T_1) = 0.2$, $P(T_2) = 0.3$, $P(T_3) = 0.5$

$P(A|T_1) = 0.7$ $P(A|T_2) = 0.4$, $P(A|T_3) = 0.3$

$$P(A) = P(A|T_1) \cdot P(T_1) + P(A|T_2) \cdot P(T_2) + P(A|T_3) \cdot P(T_3)$$

$$= 0.2 \cdot 0.7 + 0.3 \cdot 0.4 + 0.5 \cdot 0.3 =$$

$$= 0.14 + 0.12 + 0.15 = 0.41$$

$$P(T_1|A)\cdot P(A) = P(A|T_1)\cdot P(T_1)$$

$$P(T_1|A) = \frac{P(A|T_1)\cdot P(T_1)}{P(A)} = \frac{0.7\cdot 0.2}{0.41}$$

$$= \frac{14}{41}$$

$$P(T_2|A) = \frac{P(A|T_2)\cdot P(T_2)}{P(A)} = \frac{0.4\cdot 0.3}{0.41} = \frac{12}{41}$$

$$P(T_3|A) = \frac{P(A|T_3)\cdot P(T_3)}{P(A)} = \frac{0.3\cdot 0.5}{0.41} = \frac{15}{41}$$

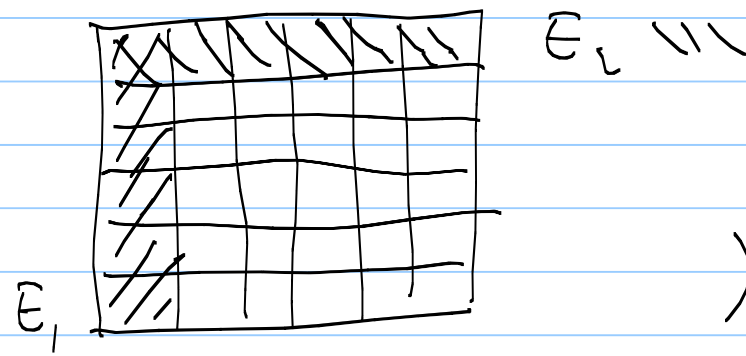
Theorem: The conditional probability is a probability 3.5 p. 93.

Definition: Two events E, F are called independent if $P(E \cap F) = P(E) \cdot P(F)$.

(Note: implicit already. Throwing two dice. The event E_1 of the first die coming up a 6 and the event E_2 of the second die coming up a 6 are independent — by equally likely outcomes

$E_1 \cap E_2 \quad = \quad 1$ out of 36 choices

$E_1, E_2 \quad = \quad 1$ out of 6 choices



<u>Problem</u>: Suppose two students take a test.
The probability that student X gets an A is 0.4
The probability student Y got an A is 0.7 and
suppose these are independent events. Assuming

we know X or Y got an A, what is the probability they both got an A?

Solution: A_1 = X got an A

A_2 = Y got an A

$P(A_1) = 0.4 \qquad P(A_2) = 0.7$

$$A(A_1 \cup A_2) = P(A_1) + P(A_2) - P(A_1 \cap A_2)$$

$$= P(A_1) + P(A_2) - P(A_1) \cdot P(A_2)$$

$$= 0.4 + 0.7 - 0.4 \cdot 0.7 =$$

$$= 1.1 - 0.28 = 0.82$$

$$P(A_1 \cap A_2 \mid A_1 \cup A_2) = \frac{P(A_1 \cap A_2)}{P(A_1 \cup A_2)} = \frac{0.28}{0.82} = \frac{28}{82} = \frac{14}{41}.$$

Example 5a: p. 94 An insurance company believe that there two risk groups. Group A is accident prone, Group B is not. (Everybody belongs to A or B). Research shows that a person in group A has a probability 0.4 of having an accident any given year. A person in group B has probability 0.2 of having an accident any given year. Group A has 30%, Group B 70%.

What is the probability a new policyholder will have an accident in their second year assuming they had an accident in the first year of their policy?

Solution: A = being in group A $\quad P(A) = 0.3$

A_1 = accident during the first year

A_2 = accident during the second year

? $P(A_2 | A_1) = P(A_2 | A_1 \cap A) P(A | A_1) + P(A_2 | A^c \cap A_1) P(A^c | A_1)$

$$P(A|A_1) = \frac{P(A_1|A)\cdot P(A)}{P(A_1)} = \underbrace{\frac{0.4\cdot 0.3}{0.26}} = \frac{6}{13}$$

$$P(A_1) = P(A_1|A)\cdot P(A) + P(A_1|A^c)\cdot P(A^c) =$$

$$= 0.4\cdot 0.3 + 0.2\cdot 0.7 = 0.26$$

$$P(A^c|A_1) = 1 - P(A|A_1) = 1 - \frac{6}{13} = \frac{7}{13}$$

$$P(A_2 | A\cap A_1) = P(A_2|A)$$

(because A_1, A_2 are independent!)

$$= 0.4$$

$$P(A_2 | A^c\cap A_1) = P(A_2|A^c) = 0.2$$

$$P(A_2|A_1) = P(A_2|A\cap A_1)\,P(A|A_1) + P(A_2|A^c\cap A_1)\,P(A^c|A_1)$$

$$= 0.4\cdot\frac{6}{13} + 0.2\,\frac{7}{13} = \frac{4}{10}\cdot\frac{6}{13} + \frac{2}{10}\cdot\frac{7}{13} =$$

$$= \frac{38}{130} = \boxed{\frac{19}{65}}$$ Answer

Discussion: $\frac{26}{100} < \frac{19}{65}$.

HW: ① 3.58 p. 106 – 107

② 3.59 p. 107

3 3.64 p. 107