MATH 425 10/3/2011

Note Title 10/3/2011

<u>Example</u>: If I have exactly two children, one is a boy who was born on a wednesday, what is the probability I have two boys.

<u>Solution</u>: $S = \underbrace{\{b,g\} \times \{1,2,\dots 7\}}_{\text{child 1}} \times \underbrace{\{b,g\} \times \{1,\dots,7\}}_{\text{child 2}}$

$|S| = 49 \cdot 4$

C = one of the children is a boy born on a wednesday

$$|C| = \left| \begin{matrix}\text{child 1 is a boy}\\ \text{born on Wednesday}\end{matrix} \right| + \left| \begin{matrix}\text{child 2 is a}\\ \text{boy born on Wed.}\end{matrix} \right|$$

$$- \left| \begin{matrix}\text{child 1 \& 2 are boys}\\ \text{born on Wed.}\end{matrix} \right| = 14 + 14 - 1 = 27$$

E = both children are boys

$$|E \cap C| = \left| \begin{matrix}\text{both boys,}\\ \text{child 1 on Wed}\end{matrix} \right| + \left| \begin{matrix}\text{both boys}\\ \text{child 2 on Wed}\end{matrix} \right|$$

$$- \left| \begin{matrix}\text{both boys}\\ \text{child 1 \& 2 on Wed}\end{matrix} \right| = 7 + 7 - 1 = 13$$

Answer: $\boxed{\dfrac{13}{27}}$

<u>Odds</u>: The odds of an event is

$$\frac{P(A)}{P(A^c)} = \frac{P(A)}{1-P(A)} . \in [0, \infty]$$

$\frac{x}{1-x}$ is a bijective map from $[0,1]$ to $[0,\infty]$

$$\boxed{\frac{x}{1-x} = z}$$

$x = z(1-x)$ $\qquad$ $x = z - zx$

$$x + zx = z \qquad x(1+z) = z$$

$$\boxed{x = \frac{z}{1+z}}$$

<u>Example</u>: If the odds are 3:2 that Michigan will beat Ohio State, what is the <u>probability</u> Michigan will win that game.

<u>Solution</u>: $$P = \frac{z}{1+z} = \frac{\frac{3}{2}}{1+\frac{3}{2}} = \frac{\frac{3}{2}}{\frac{5}{2}} = \frac{3}{5}.$$

<u>Example:</u> 5 strands of DNA left by the perpetrator

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An innocent person has the probability $\underline{\underline{10^{-5}}}$ of matching

10^6 residents

Assume a resident committed the crime.

10^4 ex-convicts probability α of having committed the crime

non-ex-convict have probability β.

$10^6 - 10^4$

$\alpha = c\beta$

A.J. Jones is an ex-convict, his DNA matches.

What is the probability, assuming the α, β numbers are correct that A. J. Jones is guilty?

<u>Solution:</u> $1 = 10^4 \alpha + (10^6 - 10^4)\beta$

$$\beta = \frac{1 - 10^4 \alpha}{10^6 - 10^4}.$$

G = A. J. Jones is guilty

M = A. J. Jones was the only ex-con. whose DNA matched.

$$P(G|M) = \frac{P(G\cap M)}{P(M)} = \frac{P(G)\,P(M|G)}{\underbrace{P(M|G)\,P(G) + P(M|G^c)\,P(G^c)}}$$

If Jones is guilty

$$P(M|G) = (1-10^{-5})^{9999}.$$

(1) $P\left(\begin{matrix}\text{all others}\\ \text{ex-cons}\\ \text{innocent.}\end{matrix} \middle| G^c\right) = \dfrac{P(\text{all ex-con. innocent})}{P(G^c)} =$

$$= \frac{1 - 10^4\alpha}{1-\alpha}$$

② $P\left(\begin{matrix}\text{no other}\\ \text{ex-con matched}\end{matrix} \;\middle|\; \begin{matrix}\text{all other}\\ \text{excons}\\ \text{are innocent}\end{matrix}\right) = \left(1-10^{-5}\right)^{9999}$

$$P(M \mid G^c) = 10^{-5} \cdot ① \cdot ② = 10^{-5}\left(\frac{1-10^{4}\alpha}{1-\alpha}\right)\left(1-10^{-5}\right)^{9999}$$

Jones matched

$P(G) = \alpha$

$$P(G \mid M) = \frac{P(G)\,P(M \mid G)}{P(M \mid G)\,P(G) + P(M \mid G^c)\,P(G^c)}$$

$$= \alpha \cdot \cancel{(1-10^{-5})^{9999}} \Big/ \left[\cancel{(1-10^{-5})^{9999}} \alpha + 10^{-5} \left(\frac{1-10^4\alpha}{\cancel{1-\alpha}} \right) \cancel{(1-10^{-5})^{9999}} \cancel{(1-\alpha)} \right]$$

$$= \frac{\alpha}{\alpha + 10^{-5}(1 - 10^4 \alpha)}$$

Example 32:

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		sides	colored	
Card 1	:	Red	Red	E_1
Card 2	:	Black	Black	E_2
Card 3	:	Red	Black.	E_3

Cards are mixed up, one card selected and put on the table. If the side facing up is red, what is the probability the other side is black?

Solution: R = the upward face of the chosen card is red.

$$P(E_3 \mid R) = \frac{P(E_3 \cap R)}{P(R)} = \frac{P(R \mid E_3)\, P(E_3)}{P(R \mid E_1) P(E_1) + P(R \mid E_2) P(E_2) + P(R \mid E_3) P(E_3)}$$

$$= \frac{\frac{1}{2} \cdot \frac{1}{3}}{1 \cdot \frac{1}{3} + 0 \cdot \frac{1}{3} + \frac{1}{2} \cdot \frac{1}{3}} = \frac{\frac{1}{6}}{\frac{1}{3} + \frac{1}{6}} =$$

$$= \frac{\frac{1}{6}}{\frac{3}{6}} = \frac{1}{3}.$$