MATH 425 10/7/2011

Note Title 10/7/2011

Test Solutions

(1) $$\sum_{k=0}^{n} x^k y^{n-k} \binom{n}{k} = (x+y)^n$$

$x = 2$, $y = 1$ gives the answer.

(2) $$\left(\overset{26}{EW} \;\middle|\; \overset{26}{NS} \right)$$

S = Where are the aces? $|S| = \binom{52}{4}$

E = East & West have all aces $|E| = \binom{26}{4}$

$$\frac{\binom{26}{4}}{\binom{52}{4}} = \frac{\cancel{26}\cdot\cancel{25}\cdot\overset{\cancel{8}}{\cancel{24}}\cdot 23\cdot 2}{\underset{\cancel{2}}{\cancel{52}}\cdot\underset{17}{\cancel{51}}\cdot\underset{\cancel{2}}{\cancel{50}}\cdot 49} = \boxed{\frac{46}{833}}$$

$$\begin{array}{r} 17 \\ \times 49 \\ \hline 153 \\ 68 \\ \hline 833 \end{array}$$

3 Some people observed

A, B are independent if and only if $P(A) = P(A|B)$

X means: X gets an A

Y means: Y gets an A

$$P(X \cap Y) = P(X) + P(Y) - P(X \cup Y) = 0.5 + 0.6 - 0.7$$

$$= 0.4$$

$$P(X) \cdot P(Y) = 0.5 \cdot 0.6 = 0.3 \quad \cancel{=}$$

not independent.

(4) A = grower A $\qquad$ B = grower B

G = good

$P(A) = 0.4 \qquad P(B) = 0.6 \qquad P(G|A) = 0.7$

$P(G|B) = 0.9$

$$P(G) = P(G|A)P(A) + P(G|B)P(B) = 0.7 \cdot 0.4 + 0.9 \cdot 0.6 = 0.82$$

$$P(A|G) = \frac{P(G|A)\cdot P(A)}{P(G)} = \frac{0.7\cdot 0.4}{0.82} = \frac{28}{82} = \underline{\underline{\frac{14}{41}}}$$

(5) $P(\text{no failure}) = 1 - \underbrace{P(\text{failure})}_{(0.4)^3} =$

$$= 1-(0.4)^3 = 0.936$$

$$\text{Answer} = \frac{600}{936} = \frac{300}{468}$$

$$0.4\cdot 0.4\cdot 0.4 = 0.064$$

$$= \frac{100}{156} = \frac{50}{78} = \underline{\underline{\frac{25}{39}}}$$

$P(\text{1 doesn't fail}) = 0.6$

Random variables are real functions on the sample space

$$f : S \to \mathbb{R}$$

such that

$f^{-1}([a,\infty))$ is an event (measurable)

These are called **measurable functions**.

A (measurable) event E determines a random variable:

$\chi_E : S \to \mathbb{R}$ is defined by

the characteristic function

$\chi_E(s) = 1$ if $s \in E$

$\chi_E(s) = 0$ if $s \notin E$.

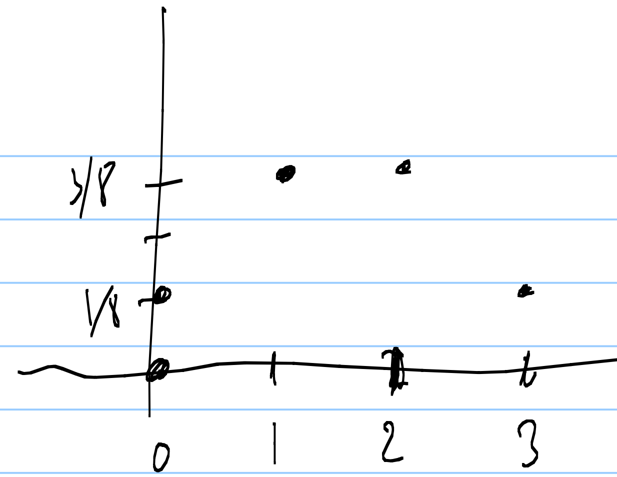
Example: Perform three independent tosses of a coin. Y = the number of times the coin came up heads. Find the probabilities of all the possible values of Y (probability distribution)

$$P(Y=0) = \frac{\binom{3}{0}}{2^3} = 1/8$$

$$P(Y=1) = \binom{3}{1}/2^3 = 3/8$$

$$P(Y=2) = \binom{3}{2}/2^3 = 3/8$$

$$P(Y=3) = \binom{3}{3}/2^3 = 1/8$$



In general, if Y = number of times a coin comes up heads in n independent tosses

$$P(Y=k) = \frac{\binom{n}{k}}{2^n}.$$

("binomial distribution")

$$= \binom{n}{k}\left(\frac{1}{2}\right)^k\left(\frac{1}{2}\right)^{n-k}$$

also allows a biased coin

where $P(\text{heads}) = p$

X = number of times the biased coin comes up heads

$$\boxed{P(X = k) = \binom{n}{k} p^k (1-p)^{n-k}} \quad \circledast$$

$$(H \mid T \mid H \mid H \mid T \cdots)$$

$$p \quad 1-p \quad p \quad p \quad 1-p$$

A discrete random variable is a random variable which can only take on countably many values.

<u>Digression into set theory</u>: A set T is <u>countable</u> if T is finite or the element of T can be ordered into a sequence.

$$T = \{ t_1, t_2, t_3, t_4, \dots t_n \mid n \in \mathbb{N} \}$$

(there exists a $\underbrace{\text{bijection}}_{\text{onto, 1-1 map}}$ $\mathbb{N} \longrightarrow T$).

<u>Example</u>: $\mathbb{Z}$ = all integers is countable.

$$\mathbb{Z} = \{0, 1, -1, 2, -2, 3, -3, \dots n, -n, \dots \}$$

Example: $\mathbb{Q}$ = all rational numbers is countable.

First count all the $\frac{k}{n}$ (k, n have no common factors)

$|k|, n \le 1$ $\qquad \frac{0}{1}, \frac{1}{1}, \frac{-1}{1}$

$|k|, n \le 2$ $\qquad \frac{1}{2}, -\frac{1}{2}$

$|k|, n \le 3$ $\qquad \frac{1}{3}, -\frac{1}{3}, \frac{2}{3}, -\frac{2}{3}$

$\vdots$

Example: $\mathbb{R}$ = all real numbers is not countable (uncountable)

All real numbers have decimal expansions, infinitely many decimals allowed, but not

$$\ldots \,.\, \ldots (k)\underbrace{9999\ldots 9\ldots}_{} \qquad k<9$$

$$\ldots = \,.\, \ldots (k+1)\,0000 \ldots 0$$

Suppose

$$\mathbb{R} = \{ a_1, a_2, a_3, a_4, \ldots a_n, \ldots \}$$

Write the decimal expansions, focus on after the decimal point digits: $a_{ij} \in \{0, \ldots, 9\}$

$a_1 = \quad ....\,. a_{11} \quad a_{12} \quad a_{13} \quad a_{14} \quad ...$

$a_2 = \quad ....\,. a_{21} \quad a_{22} \quad a_{23} \quad a_{24} \quad ...$

$a_3 = \quad ...\,. a_{31} \quad a_{32} \quad a_{33} \quad a_{34} \quad ...$

$\vdots$

I will now make a new real number

b different from all of them

$$b = 0.b_1 \, b_2 \, b_3 \, b_4 \ldots$$

If $a_{nn} = 0$ set $b_n = 1$

If $a_{nn} \neq 0$ set $b_n = 0$.

A contradiction!

HW

1. problem 4.2 p. 172
2. problem 4.3 p. 172
3. problem 4.7 p. 173