MATH 425 10/21/2011

Note Title 10/21/2011

Negative binomial distribution vs.

geometric distribution. $\leftarrow$ a special case.

$r=1$

Negative binomial = repeat independent Bernoulli trial with success probability p until we reach r failures. X = number of successes

$$P\{X=k\} = \binom{-r}{k}(-p)^k(1-p)^r$$

$$= \binom{r+k-1}{k} p^k (1-p)^r$$

Geometric variable : $r=1$ independent Bernoulli trials with success probability p

How many tries does it take until success?

$$P\{Y=k\} = (1-p)^{k-1}\,p$$

$k \geq 1$

$$= \begin{pmatrix} \text{Negative binomial variable} \\ \text{with probability } 1-p,\ r=1 \end{pmatrix} \underline{\underline{+1}}$$

Banach's matchbox problem

Banach - famous for Functional analysis

A person who smoke pipe carries two matchboxes one in the left pocket one in the right pocket.
Each matchbox initially contained N matches.
Each time the person light up, he or she reaches for a match in either box with equal probability.
When the person discovers the box he/she reached for is empty, what is the probability there will be exactly l matches in the other box left?

<u>Solution</u>: <u>Suppose</u> the <u>left</u> pocket was the one discovered empty. (The Event we are measuring & last pocket $=$ left)

Negative binomial :

"failure" = left pocket $\quad r = N+1$

"success" = right pocket $\quad k = N - \ell$

$$P\{X = k\} = \binom{-r}{k}\left(-\frac{1}{2}\right)^{k}\left(+\frac{1}{2}\right)^{N+1-k} =$$

$$= \binom{r+k-1}{k}\left(\frac{1}{2}\right)^{N+1} = \binom{2N-\ell}{N-\ell}\left(\frac{1}{2}\right)^{N+1}$$

Same probability for left pocket = right $\quad$ book

Exclusive events $\quad$ p. 159

Answer : $\quad \underline{\binom{2N-\ell}{N-\ell}\frac{1}{2^N}} = \binom{2N-\ell}{N}\frac{1}{2^N}.$

THE HYPERGEOMETRIC DISTRIBUTION

"Similar as binomial, without replacement."

Binomial: suppose we have N balls, m white
$N-m$ black,

Select a ball n times, each time throw it back.

X : how many white ones have we selected.

$$P\{X=i\} = \binom{n}{i} \Big(\underbrace{\frac{m}{N}}_{p}\Big)^{i} \Big(\underbrace{\frac{N-m}{N}}_{1-p}\Big)^{n-i}$$

__Hypergeometric__ : Exactly the same except do not throw the balls back after they are selected.

$$P\{X = i\} = \frac{\binom{n}{i}\binom{N-n}{m-i}}{\binom{N}{m}} = \frac{\binom{m}{i}\binom{N-m}{n-i}}{\binom{N}{n}}$$

N balls $\quad m$ white

$\quad n$ chosen

$X = \#(\text{white \& chosen})$

Think of N balls lined up in a row

$N = 6 \quad m = 4$

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$n = 3$ chosen #

Example: A purchaser buys set of 10 components.

8i, p. 161

test 3 at random

accepts the lot if all pass.

Suppose 30% of the lot have 4 defective component

70% have 1 defective component.

What percentage will be rejected?

Solution: A = accepts

hypergeometric

$$P(A \mid 4 \text{ defective}) \frac{3}{10} + P(A \mid 1 \text{ defective}) \frac{7}{10} =$$

$N=10,\ n=3,\ m=4$

$i=0$

$N=10,\ n=3,\ m=1$

$i=0$

$$= \frac{\binom{4}{0}\binom{6}{3}}{\binom{10}{3}} \cdot \frac{3}{10} + \frac{\binom{1}{0}\cdot\binom{9}{3}}{\binom{10}{3}} \cdot \frac{7}{10} = \frac{54}{100}$$

<u>Answer</u> : 46 %.

HW ① 4.76 p. 178–179

② 4.79 p. 179

③ 4.85 p. 179