

Math 425

12/5/2011

Note Title

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Example 5a p. 266

X, Y are jointly continuous random variables

joint density:

$$f(x,y) = \begin{cases} \frac{12}{5}x(2-x-y) & 0 < x < 1, 0 < y < 1 \\ 0 & \text{else} \end{cases}$$

Compute the conditional density of X at $X=y$,
 $0 < y < 1$.

Solution:

Computing the marginal density:

$$f(y) = \int_{-\infty}^{\infty} f(x,y) dx = \int_0^1 \frac{12}{5} x(2-x-y) dx$$

$$= \frac{12}{5} x^2 - \frac{4}{5} x^3 - \frac{6}{5} x^2 y \Big|_0^1 =$$

$$= \frac{8}{5} - \frac{6}{5}y$$

Answer: $\frac{f(x,y)}{f(y)} = \frac{\frac{12}{5}x(2-x-y)}{\frac{8}{5} - \frac{6}{5}y} = \frac{6x(2-x-y)}{4 - 3y}$.



Example 56 p. 267 Suppose X, Y have joint density

$$f(x,y) = \begin{cases} \frac{e^{-x/y} e^{-y}}{y} & 0 < x < \infty, 0 < y < \infty \\ 0 & \text{else.} \end{cases}$$

Find $P\{X > 1 \mid Y = y\}$,

Solution: Marginal density

$$f(y) = \int_0^{\infty} \frac{e^{-x/y} e^{-y}}{y} dx = -e^{-x/y} e^{-y} \Big|_0^{\infty} =$$

Conditional density: $= e^{-y}$

$$\frac{f(x,y)}{f(y)} = \frac{e^{-x/y} e^{-y}}{e^{-y}} = \frac{e^{-x/y}}{y}$$

Answer :

$$\int_1^{\infty} \frac{e^{-x/\gamma}}{\gamma} dx = -\frac{\gamma e^{-x/\gamma}}{\gamma} \Big|_{x=1}^{\infty} = \underline{\underline{e^{-1/\gamma}}}.$$

The limit theorems

Roughly speaking, the strong law of large numbers says that when we perform independent trials which are random variables with the same distribution and that the expected value exists, then the average converges to the

expected value almost certainly

To phrase the law mathematically:

We have a sample space S with a probability P .

In addition, we have a sequence of random variables $X_1, X_2, X_3, \dots, X_n, \dots$. ($X_n : S \rightarrow \mathbb{R}$).

We assume that X_n have the same cumulative distribution:

$$P\{X_n \leq a\} = P\{X_1 \leq a\} \text{ for all } a \in \mathbb{R}.$$

We assume that $E(X_1)$ ($= E(X_n)$) exists.

$$\int_S X_n dP \text{ exists } \left(\int_S |X_n| dP < \infty \right)$$

(In the discrete case: $\sum_{a \in \mathbb{R}} |a| \cdot P\{X_n = a\} < \infty$.

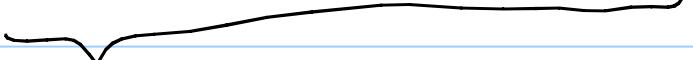
In the continuous case: $\int_{-\infty}^{\infty} x f_X(x) dx.$

We assume also that X_n are independent: If B_1, \dots, B_m are measurable subsets of S then

$$P(\{X_1 \in B_1\} \cap \{X_2 \in B_2\} \cap \dots \cap \{X_n \in B_n\}) = \\ = P\{X_1 \in B_1\} P\{X_2 \in B_2\} \cdot \dots \cdot P\{X_n \in B_n\}$$

for every n . Then

$$P \left\{ \lim_{n \rightarrow \infty} \frac{1}{n} (X_1 + \dots + X_n) = E(X_1) \right\} = 1.$$


limit exists and is equal to $E(X_i)$

Central limit theorem

If we have a sequence of identically distributed

random variables X_n which are independent such that the expected value μ exists and the variance σ^2 is finite. Then the distribution

of $\sqrt{n} \left(\left(\frac{1}{n} \sum_{i=1}^n X_i - \mu \right) \right)$ converges to the centred Gaussian with expected value 0

$$\text{HW: } \begin{array}{lll} ① 6.34 & 1.289 & ③ 6.55 \end{array}$$

$$\begin{array}{l} ② 6.39 \end{array}$$