MATH 425 12/7/2011

Note Title 12/7/2011

The last test (12/12 in class)

Continuous distributions

- Should know the definition of probability density and cumulative distribution
Compute with uniform distribution & exponential, definition of hazard rate.
- Normal (Gaussian) distribution - the formula for probability density, mean and variance

Exercises in Chapter 5 (skip dist. not covered)

- χ^2 ← PEARSON and t ← TEST OF AVERAGE (1 & 2-sided) – will provide formulas for probability density. ← more examples of statistics (internet?)

Should know the statistics story to the extent covered (if there is a question about statistics, will provide tables).

$\boxed{\text{Cauchy} = t \text{ with 1 degree of freedom}}$.

- Jointly distributed random variables – discrete, jointly continuous

  joint probability density, marginal densities

  independent jointly distributed random variables

  Exercises in Ch. 6 6.56 Th. 6.33 { transformations of jointly distributed random variables (in particular arithmetic operations on random variables) !!

multinomial distribution
multivariable Gaussian (no main axes) } Chapter 7

- Expectation is additive

  Variance is additive for independent random variables

  Covariance } Exercises & Theoretical in chapter 7 (only relevant to topics mentioned)

- What the strong law of large numbers and the central limit theorem say. } Self-test problems Chapter 9

The central limit theorem

Let $X_1, X_2, X_3, \ldots$ be a sequence of identically distributed independent random variables, each having expected value μ and variance σ^2.

Then

$$\lim_{n\to\infty} P\left\{ \frac{\sqrt{n}}{\sigma}\left(\frac{X_1 + \cdots + X_n}{n} - \mu \right) \le a \right\} = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{a} e^{-x^2/2}\, dx .$$

Example (the statistical Z test):

Measuring the distance to a star (light-years)

We want to make a series of measurements and take the average.

Outcomes (values) of measurement are independent identically distributed random variables, common mean is d (unknown) and common variance $= 4$ (light-years^2). How many measurements do we have to make to be reasonably sure $\}$ 95% we are within $\pm .5$ lightyears?

Solution:

$$Z_n = \frac{\sum_{i=1}^{n} X_i - nd}{2\sqrt{n}} \longrightarrow \text{standard Gaussian.}$$

$\sqrt{4}$

$$P\left\{-0.5 \le \frac{\sum_{i=1}^{n} X_i}{n} - d \le 0.5\right\} =$$

$$= P\left\{-0.5\,\frac{\sqrt{n}}{2} \le Z_n \le 0.5\,\frac{\sqrt{n}}{2}\right\} \approx \Phi\left(\frac{\sqrt{n}}{4}\right) - \overbrace{\Phi\left(-\frac{\sqrt{n}}{4}\right)}^{1-\Phi\left(\frac{\sqrt{n}}{4}\right)} = 2\Phi\left(\frac{\sqrt{n}}{4}\right) - 1$$

$$\Phi(a) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{a} e^{-x^2/2}\,dx$$

cummulative distribution of the standard Gaussian

$$\left.2\Phi\left(\frac{\sqrt{n}}{4}\right) - 1 = 0.95\right\} \quad \Phi\left(\frac{\sqrt{n}}{4}\right) = 0.975$$

table: $\frac{\sqrt{n}}{4} = 1.96$

$$n = (7.84)^2 \approx 61.47$$

$$\uparrow$$

$$4 \cdot 1.96$$

<u>Answer</u> : 62.