Name: _____ Score (Out of 11 points):

A non-programmable, non-scientific calculator may be used.
Let $\mathbb{F}_q$ be shorthand for the field $\mathbb{Z}/q\mathbb{Z}$ for some prime q.

1. [2 points] Which of the following codes are linear? Circle the number(s) of the linear code(s).

   i. $\Big\{(0,0,0),(1,0,0),(0,1,0),(0,0,1)\Big\} \subseteq (\mathbb{F}_2)^3$

   ii. $\Big\{(0,0,0,0),(1,1,1,1),(2,2,2,2),\ldots,\big(q-1,q-1,q-1,q-1\big)\Big\} \subseteq (\mathbb{F}_q)^4$

   iii. $\Big\{(a_1,a_2,a_3,a_4,a_5) \quad | \quad a_i \in \mathbb{F}_q, \quad a_1+a_2+a_3+a_4+a_5 \equiv 0 \pmod{q}\Big\} \subseteq (\mathbb{F}_q)^5$

   iv. $\Big\{(0,0,0),(1,2,1),(2,1,2)\Big\} \subseteq (\mathbb{F}_3)^3$

2. Let C be the $[n,k]$ linear binary code associated to the generator matrix

$$G = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \end{bmatrix}$$

   (a) [1 point] What are n and k?

   (b) [1 point] List all codewords in C.

   (c) [1 point] Given the vector $v = (1,0,1,0,1)$, write down all vectors in the coset $v + C$.

 The quiz continues on the back of the page.

(d) [2 points] Determine the minimum distance of C, and explain your reasoning.

(e) [1 point] Write down a parity check matrix for the code C.

(f) [1 point] Compute the syndrome of v.

(g) [1 point] Find a coset leader of the coset $v + C$.

(h) [1 point] Find the nearest codeword (in Hamming distance) to v.