WIM General Instructions

The Writing in the Major (WIM) project is a short essay on a mathematical topic. The paper should be typeset in LaTeX and should be about 4-7 pages in length (plus references).

The paper will be evaluated both on mathematical content and on exposition. We want to see that students have understood the mathematics underpinning their topic in some depth. We also want to see that students have put serious thought into how to present the topic in a way that is clear, mathematically rigorous (where appropriate), and engaging. The papers should be pitched at the level of a typical Math 110 student – someone familiar with the material covered so far in our course, but not necessarily familiar with the topic of the paper.

We expect that students observe the conventions for proper mathematical writing. The project is an essay: it should be written in sentences and paragraphs, with due attention paid to spelling, grammar, and writing style. It should make proper use of math environments for math typesetting, and use LaTeXsymbols properly. Math papers are usually written using first person plural ("We will prove Theorem 2.5 in three steps"), to collectively refer to both the author and the reader. Use of passive voice is acceptable too but it can be cumbersome ("Theorem 2.5 will be proved in three steps").

There are many resources available on good mathematical writing practices, with advice on (for example) how to incorporate equations and other symbolic math statements into your writing, and which shorthand is or is not considered appropriate in a formal math paper. Some suggestions: guidelines by Knuth, Larabee, and Roberts are available at `http://tex.loria.fr/typographie/mathwriting.pdf`. You're especially encouraged to read Section 1, "Minicourse on technical writing". The Math 110 handout on proof-writing is available here: `http://web.stanford.edu/~jchw/PrimerOnProof-Math110.pdf`. The Hume Center has a number of resources available to help students with their writing assignments.

We understand that students may be new to many of the conventions of writing mathematics, and encourage you to use these resources to learn more about good writing practices. This is also a reason to put a good effort into the draft submitted in February – Jeremy can give advice and corrections to mathematical writing, but he cannot meaningfully address this level of detail if there are major broader issues with structure, organization, mathematical accuracy, or English language usage.

WIM Structure. Your project should include:

A title. A good title should be specific enough to capture the content and tone of the paper. It should contain keywords (so your paper is easily found by computer-search), and, ideally, will catch the reader's attention. Include your name and the current date with the title.

An abstract. The abstract is a brief paragraph that summarizes the main results in your paper. It should appear right after the title.

An introduction. The introduction should engage the reader, motivate the content of the paper, and summarize the main results. (At the research level, the introductions to most math papers get far wider readership than the rest of the paper does, so we ought to put substantial thought into our introductions). Ideally the introduction will include some or all of:

- Motivation. Explain the importance of the subject of the paper, and its applications.

- Historical background and context for the main topic of the paper.
- An informal description of the main mathematical results of the paper.
- An example showing the main result, algorithm, or cryptosystem in action.
- A brief summary of the paper, outlining its structure and main results.

Sections. Organize the paper into numbered and titled sections, starting with the introduction. LaTeX also allows for numbered subsections (and even subsubsections).

Theorem, definition, and proof environments. State your definitions using the LaTeX definition environment, and state results using the lemma, proposition, theorem, and corollary environments. Write proofs in the proof environment. These formatting conventions help make the paper more readable, clarify its organization, and also make it easier to refer forwards or backwards to key definitions and results.

Self-contained theorem statements. Each theorem statement should be self-contained, mathematically correct and complete. This has the advantage that a reader can use your paper as a quick reference – looking directly at the statements of the theorems without having to read or re-read the whole section. It is okay to use terminology in the theorems that the reader might not know without reading the whole paper, but any variable or notation used in stating the theorem should be defined in the statement of the theorem, even if it were defined earlier in the paper. Any hypotheses should be stated explicitly, even if the same assumptions were in place in the paragraph before. (Eg., "Assume a and n are coprime".) The same holds for the statement of lemmas, propositions, and corollaries.

User-friendliness. It can take considerable thought to communicate mathematical concepts effectively. Remember that the target audience is another Math 110 student. Some strategies to consider:

- Aim to make statements simple and direct.
- Explicitly spell out details or implications that might not be obvious to someone learning the material for the first time.
- Use the LaTeX `label` and `ref` commands to let you refer to sections, definitions, theorems, and other results by number. ("Combining this result with Theorem 2.2 in Section 2, we conclude that ...")
- Break down difficult arguments into smaller steps. Modularize the arguments by making preliminary steps into lemmas or propositions. If there is a particular mathematical fact that you invoke repeatedly, make it into a numbered proposition for easy reference.
- Put thought into the notation you use. This can be as simple as choosing conventional variables – for example, it is easy for an experienced reader to remember that n denotes an integer and p a prime integer. When the arguments become more involved – with large numbers of variables, complicated functions, complicated interdependencies, and multiple indices – then decisions about how to set up notation can go far to either aid or hinder the reader's understanding of the argument.

- Use concrete examples to illustrate the main theorems or procedures. For example, if you are describing a primality test, then show how the test would apply to an actual integer, say, $n = 13697$. Or, if a proof involves a subtle argument, consider also showing how the argument applies in a concrete situation.

  It is not considered mathematically rigorous to prove a result or describe an algorithm only in the case of a particular example, but including a particular example as well as the general argument can be an excellent pedagogical technique to help the reader grasp the concept.

  Similarly, if there are certain special cases where the arguments are easier, it can be enlightening for the reader to include these (even if they are, strictly speaking, mathematically redundant). For example, "We first give the proof in the special case where the integer n is a positive prime number p ... "

- Pictures or diagrams can be powerful tools to help the reader visualize a concept or procedure. You can include pictures files (such as pdf and jpg) in LaTeX documents using the `graphicx` pacakge.

- Balance mathematical precision and readability. Mathematicians place utmost value on accuracy and precision, but this can make statements hard for the reader to decipher, especially when they are steeped in dense notation and technical hypotheses. Make hard-to-parse results more accessible by following the theorem with an informal re-phrasing of the result, a statement of the result in an easy special case, a main corollary of the result, an analogy to a more familiar result, or anything else that may give the reader perspective on 'how to think about' the statement.

Bibliography. List any books, papers, or other sources you consulted while writing your paper. Moreover, give specific references for any facts you cite or mathematical results that you state without proving.

There is a LaTeXtool called Bibtex designed to manage bibliographical information, but you may also list references by hand if this is easier.

You can use whichever citation style you choose. For example, this webpage `http://libguides.brynmawr.edu/content.php?pid=347341&sid=2893609` has some examples of how works are cited in papers published by the American Mathematics Society.

Your references list is not included in the 4-7 page suggested length.

Grading criteria

The papers will be evaluated on the basis of the points below. The most significant of these are mathematically accuracy, and content / exposition.

Mathematical correctness

- Main mathematical results are clearly and correctly stated
- Notation is defined
- No false mathematical statements
- Proofs are complete and rigorous
- Arguments are structured in a logically sound order
- Arguments do not have unnecessary or irrelevant components without reason

English language

- The paper is written in essay form, with full English sentences organized into paragraphs
- Proper spelling and grammar
- No inappropriate mixing of mathematical shorthand and English sentences
- Proper sentence structure; no run-on sentences, suitable use of first-person plural voice, etc.
- No other stylistic errors (For example, the sentence "Dividing this equation by x, the desired result follows" has a dangling participle, and should be replaced by something like "Upon dividing this equation by x, we find the desired result")
- Conventions of good mathematics writing observed; eg. English sentences do not start with a mathematical symbol. ("The integer n divides ... " instead of "n divides ...")

Mathematical typesetting

- The document is created using Latex
- Paper makes proper use of in-text and displayed mathematical environments
- Equations or mathematical statements of more than about 1 inch are displayed on their own line
- Equations are numbered and referenced by number where appropriate
- The paper uses theorem and proof environments
- The paper uses the definition environment if appropriate
- Latex symbols are used appropriately
- No typesetting problems, for example, resulting from mis-typed commands, missing backslashes, or equations running off the right side of the page

Content and exposition / mathematical clarity

- The paper covers the topics described in the instructions or another agreed-upon topic
- The project has mathematical depth, and includes a rigorous proof of a significant result
- The paper is approximately 4–7 pages, and has an appropriate amount of content for its length
- The author makes good choices of details to include
- The writing is clear and flows smoothly
- Ideas are presented in a suitable order
- The mathematical concepts are communicated effectively
- Paper is written at a level appropriate for a Math 110 classmate unfamiliar with the specific topic
- Precise definitions are given for any new or nonstandard terminology
- The paper makes good notational choices, and uses notation consistently
- Variables are declared ("Let x denote ... ") and any nonstandard notation is explained
- Pictures or diagrams are included where they clarifying
- Difficult concepts or arguments are illustrated with concrete examples
- Difficult theorem statements are re-phrased or explained in a less formal way that clarifies their meaning

Organization

- The paper has a title, author, and abstract
- The paper has an introduction
- The paper is divided into sections with meaningful titles
- Mathematical results are organized into theorems (or proposition/lemma/etc) and their proofs
- Long or complicated results are subdivided into lemmas, propositions, theorems, corollaries, etc.
- Statements such as theorems are numbered and cited by number (eg. "By Theorem 2.1, we see that ... ")
- The statement of each theorem is self-contained – the reader can read the statements of the theorems before reading the paper to get a sense of the results.
- The reader can get a good impression of the structure and content of the paper by reading section headings and theorem statements

Introduction

- The introduction gives context, history, and motivation for the problem
- The introduction includes a summary of the main result of the paper
- The introduction is engaging

Works cited

- Paper lists sources consulted
- Paper gives references for facts and unproved mathematical results

Mathematical or expository value-added

- The student has put original thought into the problem; the paper offers perspectives or insight beyond what appears in the suggested references.

Improvement of final document over draft

- The final document incorporates the grader's suggestions on the draft.