Midterm Exam

Math 490
23 October 2018
Jenny Wilson

Name: ____________________

Instructions: This exam has 4 questions for a total of 20 points.

The exam is closed-book. No books, notes, cell phones, calculators, or other devices are permitted. Scratch paper is available.

Fully justify your answers unless otherwise instructed. You may quote any results proved in class or on the homeworks without proof.

You have 80 minutes to complete the exam. If you finish early, consider checking your work for accuracy.

Jenny is available to answer questions.

Question	Points	Score
1	10	
2	3	
3	3	
4	4	
Total:	20	

1. (10 points) For each of the following, write down an example (if an example exists), or otherwise write "`Does not exist`". No further justification needed.

   (a) Metric spaces (X, d_X) and (Y, d_Y), a continuous function $f : X \to Y$, and a closed set $C \subseteq Y$ such that the **preimage** $f^{-1}(C)$ is not closed.

   (b) Metric spaces (X, d_X) and (Y, d_Y), a continuous function $f : X \to Y$, and an open set $U \subseteq X$ such that the **image** $f(U)$ is not open.

   (c) A metric space (X, d) and a nonempty set $S \subseteq X$ such that every point of S is an accumulation point of S.

   (d) A metric space (X, d), a subset $S \subseteq X$, and a point $x \in \partial S$ that is not an accumulation point of S.

   (e) A metric space (X, d) and a sequence $(a_n)_{n\in\mathbb{N}}$ in X that is convergent but not Cauchy.

   (f) A metric space (X, d) and a subset $S \subseteq X$ such that ∂S is not closed.

   (g) A metric space (X, d) that is closed and bounded, but not sequentially compact.

   (h) A metric space (X, d) that is sequentially compact, but not bounded.

   (i) A topology $(X, \mathcal{T})$, and closed sets C and D in X such that $C \cup D$ is not closed.

   (j) A topology $(X, \mathcal{T})$ that is not metrizable.

2. (3 points) Let (X, d) be a metric space, and let $(a_n)_{n\in\mathbb{N}}$ be a Cauchy sequence in X. Prove that the set $\{a_n \mid n \in \mathbb{N}\}$ is bounded.

3. (3 points) Let (X, d) be a metric space, and let $A, B \subseteq X$. Show that, if A and B are sequentially compact, then so is $A \cap B$.

4. (4 points) Let (X, d) be a metric space, and let S be a nonempty subset of X. Prove that a point $x \in X$ is in $\overline{S}$ if and only if there is a sequence of points $(s_n)_{n \in \mathbb{N}}$ in S converging to x.