Practice Midterm Exam

Math 490
October 2018
Jenny Wilson

Name: ____________________

Instructions: This exam has 4 questions for a total of 20 points.

The exam is closed-book. No books, notes, cell phones, calculators, or other devices are permitted. Scratch paper is available.

Fully justify your answers unless otherwise instructed. You may cite any results proved in class or on the homeworks without proof.

You have 80 minutes to complete the exam. If you finish early, consider checking your work for accuracy.

Jenny is available to answer questions.

Question	Points	Score
1	10	
2	2	
3	3	
4	5	
Total:	20	

1. (10 points) For each of the following, write down an example (if an example exists), or otherwise write "`Does not exist`". No further justification needed.

   (a) A metric space (X, d), a subset $Y \subseteq X$ viewed as a metric space under the restriction of the metric d, and a subset $U \subseteq Y$ that is open as a subset of Y but not open as a subset of X.

   (b) Metric spaces (X, d_X) and (Y, d_Y), a continuous function $f : X \to Y$, and an open set $U \subseteq Y$ such that $f^{-1}(U)$ is closed.

   (c) Metric spaces (X, d_X) and (Y, d_Y), a continuous function $f : X \to Y$, and a set $A \subseteq X$ such that $f(\overline{A}) \not\subseteq \overline{f(A)}$.

   (d) A metric space (X, d) and a nonempty set $S \subseteq X$ such that S is open, and S has no accumulation points.

   (e) A nonempty set $S \subseteq \mathbb{R}$ (with the Euclidean metric) such that no point contained in S is an accumulation point of S, but S has an accumulation point $x \notin S$.

   (f) A sequence of real numbers $(a_n)_{n\in\mathbb{N}}$ that is bounded but does not converge.

   (g) A sequence of real numbers $(a_n)_{n\in\mathbb{N}}$ that converges but is not bounded.

   (h) A metric space (X, d), and a subset $S \subseteq X$ that is sequentially compact but not closed.

   (i) A metric space (X, d) that is not complete.

   (j) A metric space (X, d) and a sequence $(a_n)_{n\in\mathbb{N}}$ in X such that $\{a_n \mid n \in \mathbb{N}\}$ is closed and bounded, but has no convergent subsequence.

2. (2 points) Either prove the following statement, or provide (with proof) a counterexample: Let (X, d) be a metric space, $A \subseteq X$ a subset, and $(a_n)_{n \in \mathbb{N}}$ a sequence of points in A that converge to a point a_∞. Then a_∞ is an accumulation point of A.

3. (a) (2 points) Let (X, d) be a metric space, and let $x, y \in X$. Prove that there is an open set $U \subseteq X$ such that $x \in U$ but $y \neq U$.

   (b) (1 point) Sierpinski space $\mathbb{S}$ is the set $\{0, 1\}$ with the topology $\Big\{\varnothing, \{0\}, \{0, 1\}\Big\}$. Prove that $\mathbb{S}$ is not metrizable.

4. (5 points) Let (X, d) be a metric space, and let $a_\infty \in X$. Let $(a_n)_{n\in\mathbb{N}}$ be a sequence in X with the property that every subsequence of $(a_n)_{n\in\mathbb{N}}$ has a subsequence that converges to a_∞. Prove that $(a_n)_{n\in\mathbb{N}}$ converges to a_∞.