Name: _____ Score (Out of 8 points):

1. (a) (3 points) Let X be a set. State the definition of a *metric* on X.

(b) (3 points) Define a function

$$d_\infty : \mathbb{R}^n \times \mathbb{R}^n \longrightarrow \mathbb{R}^n$$

as follows. For points $\overline{x} = (x_1, \ldots, x_n)$ and $\overline{y} = (y_1, \ldots, y_n)$ in $\mathbb{R}^n$, let

$$d_\infty(\overline{x}, \overline{y}) = \max_{i=1,\ldots,n} |x_i - y_i|.$$

Prove that d_∞ satisfies the triangle inequality.

(c) (2 points) In fact, d_∞ defines a metric on $\mathbb{R}^n$. Draw and shade the open ball $B_2(0,0)$ of radius 2 about the origin $(0,0)$ in the metric space $(\mathbb{R}^2, d_\infty)$.