

Name: _____ Score (Out of 4 points):

Definition (proper maps). Let (X, \mathcal{T}_X) and (Y, \mathcal{T}_Y) be topological spaces. A map $f : X \rightarrow Y$ is called a *proper map* if $f^{-1}(K) \subseteq X$ is compact for every compact set $K \subseteq Y$.

1. (2 points) Consider the set \mathbb{R} with the Euclidean metric. Show that the constant map

$$\begin{aligned} f : \mathbb{R} &\rightarrow \mathbb{R} \\ f(x) &= 0 \end{aligned}$$

is continuous but not proper.

2. (2 points) Let (X, \mathcal{T}_X) be a compact topological space, and let (Y, \mathcal{T}_Y) be a Hausdorff topological space. Let $f : X \rightarrow Y$ be a continuous map. Show that f is proper.

