Name: ______________________________ Score (Out of 6 points):

1. (3 points) Let $X = \{a, b, c, d\}$ be the topological space with the topology

$$\mathcal{T} = \big\{\varnothing, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, d\}, \{a, b, c\}, \{a, b, d\}, \{a, b, c, d\}\big\}.$$

Which of the following sets are bases for this topology? Circle all that apply.
**No further justification necessary.**

(i) $\mathcal{B} = \{\varnothing, X\}$

(ii) $\mathcal{B} = \{\{a\}, \{b\}, \{c\}, \{d\}\}$

(iii) $\mathcal{B} = \mathcal{T}$

(iv) $\mathcal{B} = \{\{a\}, \{b\}, \{b, c\}, \{a, d\}\}$

(v) $\mathcal{B} = \{\{a\}, \{b\}, \{a, b, c\}, \{a, b, d\}\}$

(vi) $\mathcal{B} = \{\{a, b\}, \{b, c\}, \{a, d\}\}$

2. (3 points) Let $(X, \mathcal{T})$ be a topological space, and suppose that $\mathcal{B}$ is a basis for this topology. Let $A \subseteq X$. Show that a point $x \in X$ is contained in $\overline{A}$ if and only if every basis element $B \in \mathcal{B}$ containing $x$ contains a point of $A$.