Midterm Exam

Math 490
22 October 2019
Jenny Wilson

Name: ______________________________

Instructions: This exam has 4 questions for a total of 20 points.

The exam is closed-book. No books, notes, cell phones, calculators, or other devices are permitted. Scratch paper is available.

Fully justify your answers unless otherwise instructed. You may quote any results proved in class, on a quiz, or on the homeworks without proof. Please include a complete statement of the result you are quoting.

You have 80 minutes to complete the exam. If you finish early, consider checking your work for accuracy.

Jenny is available to answer questions.

Question	Points	Score
1	9	
2	4	
3	3	
4	4	
Total:	20	

1. (9 points) For each of the following statements: if the statement is always true, write "`True`". Otherwise, state a counterexample. **No further justification needed.**

   Note: If the statement is not always true, you can receive partial credit for writing "`False`" without a counterexample.

   (a) Let $f : X \to Y$ be a continuous function of metric spaces X and Y. Then for any $A \subseteq Y$, the preimage $f^{-1}(\overline{A}) \subseteq X$ is closed.

   (b) Let X be a metric space, and $S \subseteq X$. Then $\partial S = \partial(X \setminus S)$.

   (c) Let X be a metric space, and $S \subseteq X$. Then $\partial S = \partial(\overline{S})$.

   (d) Let X be a metric space, and $A \subseteq X$. Then a point $x \in X$ is contained in $\overline{A}$ if and only if x is an accumulation point of A.

   (e) Let X and Y be metric spaces, and let $f : X \to Y$ be a continuous, invertible, open map. Then f is a homeomorphism.

(f) Every metric space is Hausdorff.

(g) Let X, Y be metric spaces, and $f : X \to Y$ a continuous function. If $S \subseteq Y$ is sequentially compact, then $f^{-1}(S)$ is sequentially compact.

(h) Every sequentially compact metric space is complete.

(i) Every complete metric space is sequentially compact.

2. (4 points) Below are two metric spaces X and subsets A. For each subset, state the interior, closure, and boundary of A, and its set A' of accumulation points. **No justification needed.**

$X = \mathbb{R}$ with the Euclidean metric, $A = \left\{ \frac{(-1)^n}{n} \;\middle|\; n \in \mathbb{N} \right\}$.

$\mathrm{Int}(A) =$ __________ $\overline{A} =$ __________ $\partial A =$ __________ $A' =$ __________

$X = \mathbb{R}$ with the discrete metric, $A = (0, 1) = \{x \in \mathbb{R} \mid 0 < x < 1\}$.

$\mathrm{Int}(A) =$ __________ $\overline{A} =$ __________ $\partial A =$ __________ $A' =$ __________

3. (3 points) Let (X, d) be a metric space, and let $A \subseteq X$. Prove that, if x is an accumulation point of A, then every neighbourhood U of x contains infinitely many points of A.

4. (4 points) Let $(X_1, d_1), (X_2, d_2), (Y_1, D_1), (Y_2, D_2)$ be metric spaces. Let $f : X_1 \to Y_1$ and $g : X_2 \to Y_2$ be continuous functions. Prove that the function

$$(f \times g) : X_1 \times X_2 \to Y_1 \times Y_2$$
$$(f \times g)(x_1, x_2) = \Big(f(x_1), g(x_2)\Big)$$

is continuous with respect to the product metrics on $X_1 \times X_2$ and $Y_1 \times Y_2$.