Name: ______________________________ Score (Out of 6 points):

1. (4 points) Fix $n \in \mathbb{N}$, and let $X = \{0,1\}^n$, the set of ordered n-tuples of 0's and 1's. We define the *Hamming distance* on X to be the function $d : X \times X \to \mathbb{R}$, where $d(x,y)$ is defined to be the number of coordinates where the n-tuples x and y differ. For example, if $n = 5$, the elements $(0,0,1,0,0)$ and $(0,0,1,1,0)$ have Hamming distance 1, because they differ in just one coordinate, the fourth, and agree elsewhere. Hamming distance is an important concept in coding theory.

   Prove that the Hamming distance d is a metric on X. Your solution should involve a complete statement of the definition of a metric.

2. (2 points) Let (X, d) be a metric space, and let $x, y \in X$ be distinct points. Prove that there exists an open ball in X that contains x but not y.

   Remark: This result shows that metric spaces are T_1-*spaces*.