

Name: \_\_\_\_\_ Score (Out of 6 points):

1. (4 points) For each of the following statements: if the statement is always true, write “True”. Otherwise, state a counterexample. **No further justification needed.**

Note: If the statement is not always true, you can receive partial credit for writing “False” without a counterexample.

(a) Let  $A$  be a subset of a topological space  $X$ . If  $A$  is connected, then  $\overline{A}$  is connected.

(b) Let  $A$  be a subset of a topological space  $X$ . If  $\overline{A}$  is connected, then  $A$  is connected.

(c) Let  $X$  be a topological space with basis  $\mathcal{B}$ . If  $X$  is disconnected, then there exist basis elements  $A, B$  in  $\mathcal{B}$  that are a separation of  $X$ .

(d) Any continuous function from  $\mathbb{R}$  (with the standard topology) to a discrete space  $X$  must be a constant function.

2. (2 points) Let  $X = \{a, b, c, d\}$  be a topological space with the topology

$$\mathcal{T} = \left\{ \emptyset, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}, \{a, b, d\}, \{a, b, c, d\} \right\}.$$

Write down a formula for a continuous path in  $X$  from  $a$  to  $d$ . **No justification necessary.**