Name: ______________________ Score (Out of 6 points):

1. (3 points) For each of the following statements: if the statement is always true, write "`True`". Otherwise, state a counterexample. **No further justification needed.**

   Note: If the statement is not always true, you can receive partial credit for writing "`False`" without a counterexample.

   (a) Let $(X, \mathcal{T}_X)$ and $(Y, \mathcal{T}_Y)$ be topological spaces, and $f : X \to Y$ a continuous function. If $C \subseteq Y$ is compact, then $f^{-1}(C)$ is compact.

   (b) Let $(X, \mathcal{T})$ be a compact topological space. Then every closed subset of X is compact.

   (c) Let $(X, \mathcal{T})$ be a compact topological space. Then every compact subset of X is closed in X.

2. (3 points) Let X be a compact space and Y a Hausdorff space. Suppose that a map $f : X \to Y$ is continuous, and has an inverse f^{-1}. Prove that f^{-1} is continuous.

   This result implies:

   > **Theorem.** A continuous bijection from a compact space to a Hausdorff space is a homeomorphism.