

Name: \_\_\_\_\_ Score (Out of 8 points):

1. (2 points) Let  $(X, d_X)$  and  $(Y, d_Y)$  be metric spaces. State what it means for a function  $f : X \rightarrow Y$  to be continuous. You may use any of our equivalent definitions.

2. (3 points) Let  $(X, d_X)$  and  $(Y, d_Y)$  be metric spaces, and let  $y_0 \in Y$ . Show that the constant function

$$\begin{aligned} f : X &\longrightarrow Y \\ f(x) &= y_0 \quad \text{for all } x \end{aligned}$$

is always continuous.

3. (3 points) Let  $(X, d_X)$  and  $(Y, d_Y)$  be metric spaces. Let  $S$  be a subset of  $X$ . Recall that  $S$  inherits a metric space structure under restriction of the metric on  $X$ .

Let  $f : X \rightarrow Y$  be a function. Recall that the *restriction* of  $f$  to  $S$ , often written  $f|_S$ , is the function

$$\begin{aligned} f|_S : S &\longrightarrow Y \\ f|_S(s) &= f(s). \end{aligned}$$

Prove that, if  $f$  is a continuous function, then its restriction  $f|_S$  to  $S$  is also a continuous function.