Name: ______________________________ Score (Out of 9 points):

1. (6 points) Which of the following functions are continuous? Briefly explain your answer.

(a)

$$f : (\mathbb{R}, \text{Euclidean}) \to (\mathbb{R}, \text{cofinite})$$
$$f(x) = x$$

(b)

$$f : (\mathbb{R}, \text{cofinite}) \to (\mathbb{R}, \text{cofinite})$$
$$f(x) = \sin(x)$$

(c) Let $X = \{a, b\}$ with the topology $\mathcal{T} = \Big\{\varnothing, \{a\}, \{a, b\}\Big\}$.

$$f : X \to X$$
$$f(a) = b$$
$$f(b) = a$$

2. (3 points) Our concept of an accumulation point also makes sense in abstract topological spaces:

   Definition. Let $(X, \mathcal{T})$ be a topological space, and $A \subseteq X$. A point $x \in X$ is an *accumulation point* of A if and only if every neighbourhood of x contains a point of A distinct from x.

   Let $(X, \mathcal{T})$ be a topological space, and $A \subseteq X$. Let $\mathcal{B}$ be a basis for $\mathcal{T}$. Prove that x is an accumulation point of A if and only if every basis element $B \in \mathcal{B}$ containing x contains a point of A distinct from x.