Name: ______________________________ Score (Out of 9 points):

1. (6 points) Consider the following topological spaces X and their subsets S. In each case, compute the interior $\mathrm{Int}(S)$, the closure $\overline{S}$, the boundary ∂S, and the set S' of accumulation points of S. **No justification necessary.**

   (a) Let $X = \{a, b, c, d\}$ with the topology $\Big\{\varnothing, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}, \{a, b, c, d\}\Big\}$, $S = \{a, c\}$.

   $\mathrm{Int}(S)$: __________ $\overline{S}$: __________ ∂S: __________ S': __________

   (b) Let $S = (-\infty, 0) \cup \{1\} \subseteq \mathbb{R}$, where $\mathbb{R}$ has the topology $\{(a, \infty) \mid a \in \mathbb{R}\} \cup \{\varnothing\} \cup \{\mathbb{R}\}$.

   $\mathrm{Int}(S)$: __________ $\overline{S}$: __________ ∂S: __________ S': __________

   (c) Let $S = \{0, 1\} \subseteq \mathbb{R}$, where $\mathbb{R}$ has the cofinite topology.

   $\mathrm{Int}(S)$: __________ $\overline{S}$: __________ ∂S: __________ S': __________

2. (2 points) For each of the following statements: if the statement is always true, write "`True`". Otherwise, state a counterexample. **No further justification needed.**

   Note: If the statement is not always true, you can receive partial credit for writing "`False`" without a counterexample.

   (a) Let A be a subset of a topological space X. Then A is closed if and only if it contains its boundary.

   (b) Let A be a subset of a topological space X. Then $\partial(\partial A) = \partial A$.