Name: ______________________ Score (Out of 8 points):

1. (6 points) Consider the following topological spaces X and their subsets S. In each case, compute the interior $\mathrm{Int}(S)$, the closure $\overline{S}$, the boundary ∂S, and the set S' of accumulation points of S. **No justification necessary.**

   (a) Let $X = \{a, b, c, d\}$ with the topology $\Big\{\varnothing, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}, \{a, b, c, d\}\Big\}$, $S = \{a, c\}$.

   $\mathrm{Int}(S)$: $\{a\}$ $\overline{S}$: $\{a, c, d\}$ ∂S: $\{c, d\}$ S': $\{c, d\}$

   (b) Let $S = (-\infty, 0) \cup \{1\} \subseteq \mathbb{R}$, where $\mathbb{R}$ has the topology $\{(a, \infty) \mid a \in \mathbb{R}\} \cup \{\varnothing\} \cup \{\mathbb{R}\}$.

   $\mathrm{Int}(S)$: $\varnothing$ $\overline{S}$: $(-\infty, 1]$ ∂S: $(-\infty, 1]$ S': $(-\infty, 1)$

   (c) Let $S = \{0, 1\} \subseteq \mathbb{R}$, where $\mathbb{R}$ has the cofinite topology.

   $\mathrm{Int}(S)$: $\varnothing$ $\overline{S}$: $\{0,1\}$ ∂S: $\{0,1\}$ S': $\varnothing$

2. (2 points) For each of the following statements: if the statement is always true, write "`True`". Otherwise, state a counterexample. **No further justification needed.**

   Note: If the statement is not always true, you can receive partial credit for writing "`False`" without a counterexample.

   (a) Let A be a subset of a topological space X. Then A is closed if and only if it contains its boundary.

   True. *Hint:* You have proved that A is closed if and only if $A = \overline{A}$. The equation $\partial A = \overline{A} \setminus \mathrm{Int}(A)$ implies that $\overline{A} = \mathrm{Int}(A) \cup \partial A$. Use this to argue that $A = \overline{A}$ if and only if A contains its boundary.

   (b) Let A be a subset of a topological space X. Then $\partial(\partial A) = \partial A$.

   False. For example, let $X = \mathbb{R}$ with the Euclidean topology, and $A = \mathbb{Q}$.
   Then $\partial \mathbb{Q} = \mathbb{R}$, but $\partial(\partial \mathbb{Q}) = \partial \mathbb{R} = \varnothing$.