

Administrative

1. Review the information on the course webpage

<http://www.math.lsa.umich.edu/~jchw/2019Math590.html>

under **Course Information**. Please pay particular attention to the homework policy, and to the midterm and final exam dates. Contact Jenny (jchw@umich.edu) if you have any questions.

2. If you would like to be able to attend Jenny's office hours, indicate on this Doodle poll when you are available: <https://doodle.com/poll/3vhbrkh3krt74m7u>. (Although the poll asks for dates/times on a specific week, please only select a time if you expect to be available at this time and day of the week regularly throughout the semester).
3. If you might need an accommodation in the class based on the impact of a disability, please get in touch with Jenny at jchw@umich.edu. You may be asked to obtain a VISA form through the Office of Services for Students with Disabilities (SSD).
4. Please contact Jenny as soon as possible if you anticipate a conflict with the midterm or final exam.

Assignment questions

(Hand these questions in!)

1. (**Functions review.**) Let $f : X \rightarrow Y$ be a function of sets X and Y . Recall that, for $A \subseteq X$, the *image* of A under f is the subset of Y

$$f(A) = \{f(a) \in Y \mid a \in A\} \subseteq Y.$$

For $C \subseteq Y$, the *preimage* of C under f is the subset of X

$$f^{-1}(C) = \{c \in X \mid f(c) \in C\} \subseteq X.$$

Let $A, B \subseteq X$ and $C, D \subseteq Y$. For each of the following, determine whether you can replace the symbol \square with \subseteq , \supseteq , $=$, or none of the above. You should understand how to justify your solution, but for this homework problem, you only need to submit the final answer (\subseteq , \supseteq , $=$, or "none of the above").

- (a) $f(A \cap B) \square f(A) \cap f(B)$ (b) $f(A \cup B) \square f(A) \cup f(B)$
- (c) For $A \subseteq B$, $f(B \setminus A) \square f(B) \setminus f(A)$
- (d) $f^{-1}(C \cup D) \square f^{-1}(C) \cup f^{-1}(D)$ (e) $f^{-1}(C \cap D) \square f^{-1}(C) \cap f^{-1}(D)$
- (f) For $C \subseteq D$, $f^{-1}(D \setminus C) \square f^{-1}(D) \setminus f^{-1}(C)$
- (g) $A \square f^{-1}(f(A))$ (h) $C \square f(f^{-1}(C))$

2. (**Cartesian product review.**) For sets X and Y , let $A, B \subseteq X$ and $C, D \subseteq Y$. Consider the Cartesian product $X \times Y$. For each of the following equalities, either prove the equality holds for all possible sets, or give a counterexample.

- (a) $(A \times C) \cup (B \times D) = (A \cup B) \times (C \cup D)$
- (b) $(A \times C) \cap (B \times D) = (A \cap B) \times (C \cap D)$
- (c) $(X \setminus A) \times (Y \setminus C) = (X \times Y) \setminus (A \times C)$