Midterm Exam

Math 590
13 March 2019
Jenny Wilson

Name: ____

Instructions: This exam has 4 questions for a total of 20 points.

The exam is closed-book. No books, notes, cell phones, calculators, or other devices are permitted. Scratch paper is available.

Fully justify your answers unless otherwise instructed. You may quote any results proved in class or on the homeworks without proof, but please include a precise statement of the result you are quoting.

You have 50 minutes to complete the exam. If you finish early, consider checking your work for accuracy.

Jenny is available to answer questions.

Question	Points	Score
1	7	
2	8	
3	2	
4	3	
Total:	20	

1. (7 points) Each of the following statements is either true or false. If the statement holds in general, circle "**T**". Otherwise, circle "**F**". **No justification necessary.**

   (a) For sets A, B, C, D, there is equality $(A \times B) \cup (C \times D) = (A \cup C) \times (B \cup D)$. **T** **F**

   (b) The set of all functions $\{f : \mathbb{N} \to \mathbb{N}\}$ is countable. **T** **F**

   (c) Consider $\mathbb{R}$ with the cofinite topology. Let $f : \mathbb{R} \to \mathbb{R}$ be the function given by $f(x) = \sin(x)$. Then f is continuous. **T** **F**

   (d) Let X be a metric space and $B_r(x)$ a ball of radius r in X. Then any two points in $B_r(x)$ are distance less than $2r$ apart. **T** **F**

   (e) Let X and $X_i, (i \in I)$, be topological spaces. A function $f : X \to \prod_{i \in I} X_i$ is continuous with respect to the product topology on $\prod_{i \in I} X_i$ if and only if all of its coordinate functions are continuous. **T** **F**

   (f) Let X be a metric space, and let Y be a bounded metric space. If X is homeomorphic to Y (with respect to the metric topologies), then X is also bounded. **T** **F**

   (g) The set $\mathbb{R}$ with the cofinite topology is connected. **T** **F**

2. (8 points) Each of the following statements is either true or false. If the statement holds in general, write "`True`". Otherwise, state a counterexample. **No justification necessary.** You can get partial credit for correctly writing "`False`" without a counterexample.

   (a) If C is a closed set, then C is the closure of some open set U.

   (b) Suppose that $(a_n)_{n\in\mathbb{N}}$ is a convergent sequence in a topological space X, and $f : X \to Y$ is a continuous function. Then $\Big(f(a_n)\Big)_{n\in\mathbb{N}}$ is a convergent sequence in Y.

   (c) Let $\{X_i\}_{i\in I}$ be a collection of T_1-spaces. Then the product topology on $\prod_{i\in I} X_i$ has the T_1 property.

   (d) Let X be a topological space and A a subspace. If C is closed in A, then C is closed in X.

   (e) Let $f : \mathbb{R} \to Y$ be a continuous function from $\mathbb{R}$ (with the standard topology) to a Hausdorff space Y. Then f is completely determined by its values on $\mathbb{Q} \subseteq \mathbb{R}$.

   (f) Let X be a topological space with the property that a sequence $(a_n)_{n\in\mathbb{N}}$ converges in X if and only if the sequence is eventually constant. (Recall that this means that there is some $N \geq 0$ so that $a_n = a_N$ for all $n \geq N$.) Then X has the discrete topology.

   (g) Let $p : X \to A$ be a quotient map. If X is metrizable, then A is metrizable.

   (h) Let A and B be nonempty subsets of a topological space. If $A \cap B = \varnothing$, then $A \cup B$ is disconnected.

3. (2 points) Consider the standard topology on $\mathbb{R}$. Recall that $\mathbb{R}^\omega$ denotes the space $\prod_{\mathbb{N}} \mathbb{R}$ of sequences of real numbers. Let $X \subseteq \mathbb{R}^\omega$ be the set of all sequences that are eventually zero. (Recall that $(a_n)_{n\in\mathbb{N}}$ is *eventually zero* if there is some $N \geq 0$ so that $a_n = 0$ for all $n \geq N$.) State the following. **No justifiication needed.**

   Closure of X in product topology: ____________________

   Closure of X in box topology: ____________________

4. (3 points) Let $p : X \to Y$ be a continuous map. Suppose that there exists a continuous map $f : Y \to X$ such that $p \circ f$ is the identity function on Y. Prove that p is a quotient map.

 End of exam.