Name: ______________________________ Score (Out of 9 points):

1. (6 points) State which of the following sets is countable by circling either "`Countable`" or "`Uncountable`". No justifcation necessary.

• The set $\mathbb{Q} \times \mathbb{Q} \times \mathbb{Q}$	`Countable`	`Uncountable`
• The set of all finite subsets of $\mathbb{Q}$	`Countable`	`Uncountable`
• The set of irrational numbers $\mathbb{R} \setminus \mathbb{Q}$	`Countable`	`Uncountable`
• The set of all sequences $(a_n)_{n \in \mathbb{N}}$ with $a_n \in \mathbb{Z}$ for all n	`Countable`	`Uncountable`
• The set of monic polynomials with integer coefficients $\{x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0 \mid a_i \in \mathbb{Z}, n \in \mathbb{N}\}$	`Countable`	`Uncountable`
• The set of all real numbers x that can be represented by a decimal expansion involving only the digits 0 and 1	`Countable`	`Uncountable`

2. (3 points) Let $S = \{0, 1\}$ and let S^ω denote the Cartesian product

$$S^\omega = S \times S \times S \times S \times \cdots$$

of a countably infinite number of copies of S. (So an element of S^ω is an ordered tuple such as

$$(0, 0, 1, 0, 1, 1, 1, 0, 1, 0, 1, 1, 0, 0, 0, \cdots)\Big).$$

Show that S^ω is not countable.