Name: ______________________________ Score (Out of 7 points):

1. (3 points) Let X be a set and let $p \in X$. Prove that the following is a topology on X:

$$\mathcal{T} = \{X\} \cup \{U \subseteq X \mid p \notin U\}.$$

2. (4 points) Let $n \in \mathbb{N}$ and let $(X_1, \mathcal{T}_1), (X_2, \mathcal{T}_2), \ldots, (X_n, \mathcal{T}_n)$ be topological spaces. Consider the topology $\mathcal{T}$ on $X_1 \times X_2 \times \cdots \times X_n$ generated by the basis

$$\mathcal{B} = \{U_1 \times U_2 \times \cdots \times U_n \mid U_i \in \mathcal{T}_i \text{ for all } i\}.$$

(You do not need to show that this is a basis.) Let $(X, \mathcal{T}_X)$ be a topological space, and for each $i = 1, \ldots, n$ let $f_i : X \to X_i$ be a function. Define

$$\begin{aligned} f : X &\longrightarrow X_1 \times X_2 \times \cdots \times X_n \\ x &\longmapsto (f_1(x), f_2(x), \ldots, f_n(x)) \end{aligned}$$

Show that the function f is continuous (with respect to the topologies $\mathcal{T}_X$ and $\mathcal{T}$) if and only if each coordinate function f_i is continuous (with respect to the topologies $\mathcal{T}_X$ and $\mathcal{T}_i$).