Name: ____________________ Score (Out of 9 points):

1. (6 points) Compute the interior, closure, boundary, and set of all limit points of the following sets S. **No justification necessary.**

   (a) Let $X = \{a, b, c, d\}$ with the topology $\Big\{\varnothing, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}, \{a, b, c, d\}\Big\}$, $S = \{b, c\}$.

Int(S): $\{b\}$ $\overline{S}$: $\{b, c, d\}$ ∂S: $\{c, d\}$ Limit points: $\{c, d\}$

   (b) Let $S = (-\infty, 0) \cup \{1\} \subseteq \mathbb{R}$, where $\mathbb{R}$ has the topology $\{(a, \infty) \mid a \in \mathbb{R}\} \cup \{\varnothing\} \cup \{\mathbb{R}\}$.

Int(S): $\varnothing$ $\overline{S}$: $(-\infty, 1]$ ∂S: $(-\infty, 1]$ Limit points: $(-\infty, 1)$

   (c) Let $S = \{0, 1\} \subseteq \mathbb{R}$, where $\mathbb{R}$ has the topology $\{U \mid 0 \in U\} \cup \{\varnothing\}$.

Int(S): $\{0,1\}$ $\overline{S}$: $\mathbb{R}$ ∂S: $\mathbb{R}\backslash\{0,1\}$ Limit points: $\mathbb{R}\backslash\{0\}$

2. (3 points) Each of the following statements is either true or false. If the statement holds in general, write "`True`". Otherwise, state an example of a topological space X and a subset S for which it fails. **No justification necessary.**

   Note: You can get partial credit for correctly writing "`False`" without a counterexample.

   (a) Let $(X, \mathcal{T})$ be a topological space, and $S \subseteq X$. Then every point of $\mathrm{Int}(S)$ is a limit point of S.

   False. For example, consider the topological space $X = \{0, 1\}$ with the discrete topology, and the subset $S = \{0\}$. Then 0 is an interior point, but not a limit point, of S.

   (b) Let $(X, \mathcal{T})$ be a topological space, and $S \subseteq X$. Then $\overline{S} = \overline{\mathrm{Int}(S)}$.

   False. For example, $\mathbb{R}$ with the standard topology, and the subset $S = \mathbb{Q}$.
   Then $\overline{\mathbb{Q}} = \mathbb{R}$, but $\overline{\mathrm{Int}(\mathbb{Q})} = \overline{\varnothing} = \varnothing$.

   (c) Let $(X, \mathcal{T})$ be a topological space, and $S \subseteq X$. Then $\partial(\partial S) = \partial S$.

   False. For example, $\mathbb{R}$ with the standard topology, and the subset $S = \mathbb{Q}$.
   Then $\partial \mathbb{Q} = \mathbb{R}$, but $\partial(\partial \mathbb{Q}) = \partial \mathbb{R} = \varnothing$.