

Name: \_\_\_\_\_ Score (Out of 8 points):

1. (4 points) Find the set of all limits of the following sequences. If the sequences does not converge to any point, write "Does not converge". **No justification necessary.**

- Let  $X = \{a, b, c, d\}$  have the topology  $\{\emptyset, \{a\}, \{b\}, \{a, b\}, \{a, b, c, d\}\}$ .

(i)  $a, b, a, b, a, b, a, b, \dots$

- Let  $\mathbb{R}$  have the topology  $\mathcal{T} = \{(a, \infty) \mid a \in \mathbb{R}\} \cup \{\emptyset\} \cup \{\mathbb{R}\}$ .

(ii)  $0, 0, 0, 0, 0, 0, 0, 0, \dots$

(iii)  $(-n)_{n \in \mathbb{N}}$

- Let  $\mathbb{R}$  have the topology  $\mathcal{T} = \{\emptyset\} \cup \{U \subseteq \mathbb{R} \mid 0 \in U\}$ .

(iv)  $0, 0, 0, 0, 0, 0, 0, 0, \dots$

2. (4 points) Show that a topological space  $X$  is Hausdorff if and only if, for each  $x \in X$ ,

$$\bigcap_{U \text{ a neighbourhood of } x} \bar{U} = \{x\}.$$