

Name: _____ Score (Out of 7 points):

1. (3 points) Let $X = \{a, b, c\}$. For each of the following topologies on X , write down a path from a to c , if one exists. Otherwise, write "no path exists".

(a) $\mathcal{T} = \{\emptyset, \{a\}, \{b, c\}, \{a, b, c\}\}$

(b) $\mathcal{T} = \{\emptyset, \{a\}, \{a, b\}, \{a, b, c\}\}$

(c) $\mathcal{T} = \{\emptyset, \{a\}, \{c\}, \{a, c\}, \{a, b, c\}\}$

2. (4 points) Prove the following result.

Theorem. A space X is locally path-connected if and only if for every open set U of X , each path component of U is open in X .