

Name: _____ Score (Out of 12 points):

1. (8 points) State which of the following topologies on \mathbb{R} are “first countable”, “second countable”, or neither, by circling the appropriate term(s). No justification necessary.

- $(\mathbb{R}, \text{standard topology})$
first countable second countable
- $(\mathbb{R}, \text{discrete topology})$
first countable second countable
- $(\mathbb{R}, \text{indiscrete topology})$
first countable second countable
- $(\mathbb{R}, \{(a, \infty) \mid a \in \mathbb{R}\} \cup \{\emptyset\} \cup \{\mathbb{R}\})$
first countable second countable
- $(\mathbb{R}, \{U \mid 0 \notin U\} \cup \{\mathbb{R}\})$
first countable second countable
- $(\mathbb{R}, \text{cofinite})$
first countable second countable
- $(\mathbb{R}, \text{cocountable})$
first countable second countable
- $(\mathbb{R}, \text{basis } \{[a, b) \mid a, b \in \mathbb{R}\})$
first countable second countable

2. (4 points) Let X be a compact metric space. Prove that X is second countable.