Name: _____      Score (Out of 4 points):

1. On this quiz we will prove the following theorem.

   **Theorem.** Let $f : X \to Y$ be a function of metric spaces. Then $f$ is continuous if and only if the preimage of every open ball in $Y$ is open. In other words, $f$ is continuous if and only if $f^{-1}\big(B_r(y)\big)$ is an open subset of $X$ for every $y \in Y$ and every $r > 0$.

   Let $f : X \to Y$ be a function between metric spaces.

   (a) (1 point) Suppose that $f$ is continuous. Explain why the preimage $f^{-1}\big(B_r(y)\big)$ of a ball is an open subset of $X$ for every $y \in Y$ and every $r > 0$.

   (b) (3 points) Suppose the function $f$ has the property that $f^{-1}\big(B_r(y)\big)$ is an open subset of $X$ for every $y$ in $Y$ and $r > 0$. Prove that $f$ is continuous.