

Name: \_\_\_\_\_ Score (Out of 5 points):

1. (3 points) Let  $(X, d)$  be a metric space, and let  $A, B \subseteq X$ . Show that, if  $A$  and  $B$  are sequentially compact, then so is  $A \cap B$ .

2. Consider the metric space  $X = (0, \infty) \subseteq \mathbb{R}$  with the Euclidean metric. For the following questions, simply state an example. **No justification needed.**

(a) (1 point) Find a sequence  $(a_n)_{n \in \mathbb{N}}$  in  $X$  that is Cauchy, and a continuous function  $f: X \rightarrow X$  such that the sequence  $(f(a_n))_{n \in \mathbb{N}}$  is not Cauchy.

(b) (1 point) Find a sequence  $(a_n)_{n \in \mathbb{N}}$  in  $X$  that is unbounded, and a continuous function  $f: X \rightarrow X$  such that the sequence  $(f(a_n))_{n \in \mathbb{N}}$  is bounded.