Name: ______________________ Score (Out of 5 points):

1. (2 points) $X = \{a, b, c\}$, $\mathcal{T} = \Big\{\varnothing, \{c\}, \{b, c\}, \{a, b, c\}\Big\}$, $A = \{a, c\}$.

$\text{Int}(A) =$ $\{c\}$ $\overline{A} =$ $X = \{a, b, c\}$ $\partial A =$ $\{a, b\}$ $A' =$ $\{a, b\}$

2. (3 points) Let A be a subset of a topological space X. Prove that $X \setminus \overline{A} = \text{Int}(X \setminus A)$.

Solutions.

Our solution will use the following definitions:

The *interior* of a set B is the set of interior points of B. A point $x \in B$ is an *interior point* of B if and only if x has an open neighbourhood contained in B.

A point $x \in X$ is in the *closure* of a set A if and only if every open neighbourhood of x contains a point of A. Thus, a point $x \in X$ is not in the closure of A if and only if there exists some open neighbourhood of x that is disjoint from A.

Putting these definitions together:

$$\begin{aligned}
\text{Int}(X \setminus A) &= \{x \in X \mid x \text{ is an interior point of } X \setminus A\} \\
&= \{x \in X \mid \text{ there is an open neighbourhood } U \text{ of } x \text{ with } x \in U \subseteq X \setminus A\} \\
&= \{x \in X \mid \text{ there is an open neighbourhood } U \text{ of } x \text{ with } U \cap A = \varnothing\} \\
&= \{x \in X \mid x \notin \overline{A}\} \\
&= X \setminus \overline{A}.
\end{aligned}$$