

Name: _____

Score (Out of 6 points):

1. (6 points) State whether each of the following functions is continuous and/or open by circling “Continuous” and/or “Open”. No justification necessary.

$$f : (\mathbb{R}, \text{cofinite}) \rightarrow (\mathbb{R}, \text{Euclidean})$$

$$f(x) = x$$

Continuous

Open

$$g : (\mathbb{R}, \text{discrete}) \rightarrow (\mathbb{R}, \text{Euclidean})$$

$$g(x) = x$$

Continuous

Open

$$k : (\mathbb{R}, \text{indiscrete}) \rightarrow (\mathbb{R}, \text{Euclidean})$$

$$k(x) = x$$

Continuous

Open

Let $X = \mathbb{R}$ and $\mathcal{T} = \{(a, \infty) \mid a \in \mathbb{R}\} \cup \{\mathbb{R}\} \cup \{\emptyset\}$.

$$h : (\mathbb{R}, \mathcal{T}) \rightarrow (\mathbb{R}, \text{cofinite})$$

$$h(x) = x$$

Continuous

Open

Let X, Y be any topological spaces, and endow $X \times Y$ with the product topology. Let π_X be the projection map

$$\pi_X : X \times Y \rightarrow X$$

$$\pi_X(x, y) = x$$

Continuous

Open

Let $X = \{a, b, c, d\}$ and

$$\mathcal{T} = \{\emptyset, \{a, b\}, \{c\}, \{a, b, c\}, \{a, b, c, d\}\}.$$

Continuous

Open

$$F : (\mathbb{R}, \text{Euclidean}) \rightarrow (X, \mathcal{T})$$

$$F(x) = \begin{cases} a, & x \in (0, 1] \\ b, & x \in (1, \infty) \\ c, & x \in (-\infty, 0) \\ d, & x = 0. \end{cases}$$
