Midterm Exam II

Math 592
18 March 2021
Jenny Wilson

Name: ______________________________

Instructions: This exam has 4 questions for a total of 15 points.

The exam is **closed-book**. No books, notes, cell phones, calculators, or other devices are permitted.

Fully justify your answers unless otherwise instructed. You may quote any results proved in class, on a quiz, or on the homeworks without proof. Please include a complete statement of the result you are quoting.

You have 60 minutes to complete the exam. If you finish early, consider checking your work for accuracy.

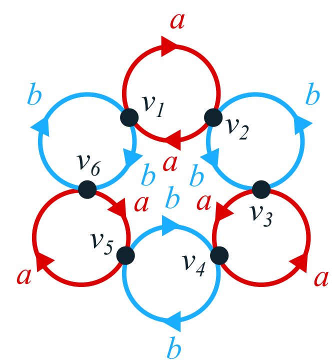
Jenny is available to answer questions.

Question	Points	Score
1	4	
2	4	
3	3	
4	4	
Total:	15	

Notation

- $I = [0, 1]$ (closed unit interval)
- $D^n = \{x \in \mathbb{R}^n \mid |x| \leq 1\}$ (closed unit n-disk)
- $S^n = \partial D^{n+1} = \{x \in \mathbb{R}^{n+1} \mid |x| = 1\}$ (unit n-sphere) (we may view S^1 as the unit circle in $\mathbb{C}$)
- $S^\infty = \bigcup_{n\geq 1} S^n$ with the weak topology
- Σ_g closed genus-g surface
- $\mathbb{R}\mathrm{P}^n$ real projective n-space
- $\mathbb{C}\mathrm{P}^n$ real complex n-space

1. (4 points) Identify $\pi_1(S^1 \vee S^1)$ with the free group F_2 on a, b in our conventional way. Consider the following cover of $p : X \to S^1 \vee S^1$. Answer each of the following. **No justification necessary.**



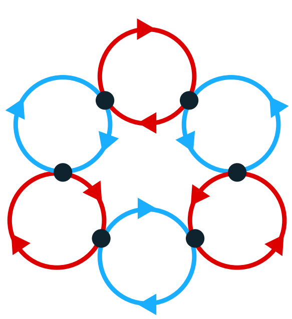
(a) State a free generating set for $H = p_*(\pi_1(X, v_1))$.

(b) Circle one: this cover is … `REGULAR` `NOT REGULAR`

(c) State the deck group for this cover (either by name or by a group presentation).

(d) Let x_0 be the wedge point in $S^1 \vee S^1$. The elements in the fibre over x_0 are labelled $v_1, v_2, \ldots v_6$. State the action of $a \in F_2$ on the fibre as a permutation of these 6 vertices.

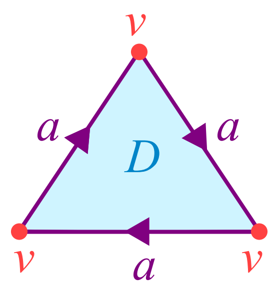
(e) Is $a \in F_2$ in the normalizer of $H = p_*(\pi_1(X, v_1))$? If so, describe the deck transformation of (X, v_1) defined by a (using the picture below to illustrate). If not, write "`No deck map exists`".



2. (4 points) (a) Let D^2 be the closed 2-disk. Suppose G is a group acting on D^2 by a covering space action. Prove that G must be the trivial group.

(b) Let X be a path-connected, locally path-connected space. Prove that any cover $D^2 \to X$ must be a homeomorphism.

3. (3 points) Compute the simplicial homology groups $H_n(X)$ of the following Δ-complex X. Describe each homology group in the sense of the structure theorem for finitely generated abelian groups.



4. (4 points) For each of the following statements: if the statement is true, write "`True`". Otherwise, state a counterexample. **No further justification needed.**

   Note: If the statement is not true, you can receive partial credit for writing "`False`" without a counterexample.

   (a) Every continuous map from $\mathbb{R}\mathrm{P}^2$ to an n-torus $(S^1)^n$ is nullhomotopic.

   (b) Let X be a path-connected, locally path-connected space. Then X has a universal cover.

   (c) Let $\tilde{X} \to X$ be a (not necessarily connected) covering space, and let τ be a deck map $\tilde{X} \to \tilde{X}$. If τ fixes a point, then τ is the identity.

   (d) Let F_2 be the free group on 2 letters a, b. Then any finitely-generated nontrivial subgroup of F_2 has finite index.