1 Math 592 Prerequisites

Math 592 is an intensive introductory class on algebraic topology. I will assume students are fluent with the foundations of point-set topology, basic manifold theory, group theory, and theory of R-modules. Math 591 or equivalent is recommended, and Math 593 or equivalent is strongly recommended.

Required background

Students in this class are expected to be comfortable with the following concepts and the relevant foundational results. This list is probably not exhaustive.

Set theory and functions

- injective, surjective, and bijective functions; images and preimages
- equivalence relations

Linear algebra

- vector spaces and subspaces
- matrices, linear maps, images and kernels
- determinant, trace
- eigenvalues, eigenvectors, diagonalization
- dual vector spaces, dual of a linear map

Complex numbers

- basic properties of $\mathbb{C}$, functions $\mathbb{C} \to \mathbb{C}$

Topology

- topological space, open and closed sets
- interior, closure, and boundary of a set
- topology induced by a metric
- continuous maps, homeomorphisms
- subspace topology
- product topology
- quotient topology
- compactness
- connectedness, path-connectedness
- Hausdorff space

Group theory

- group, subgroup, abelian group
- group homomorphism, kernel and image
- left/right cosets and quotient groups, normal subgroups
- isomorphism theorems
- direct products of groups
- conjugation
- group action, orbit, stabilizer
- orbit-stabilizer theorem
- free action, transitive action
- generators of a group

Manifold theory

- topological manifold, coordinate chart, atlas
- dimension of a manifold
- smooth manifold
- tangent space
- vector field
- surface
- orientable manifold

R-modules

- R-modules, submodules
- R-linear maps
- sums and direct sums
- free R-module on a set S
- linear independence, bases
- examples of objects defined by universal properties

Strongly recommended background

It is strongly recommended that students be familiar with the following concepts and their basic properties. Students who do not know one or more of these concepts should be prepared to commit additional time for independent study on these topics.

Group theory

- free group
- group presentation, generators and relations
- commutator subgroup, abelianization
- topological groups, Lie groups

Category theory

- category, object, morphism, + examples
- functor

R-modules

- Smith normal form
- short exact sequences, extensions
- splitting of a short exact sequence
- tensor product of a right- and left-R-module

Topology

- space-filling curves
- compact-open topology