Name: _____ Score (Out of 4 points):

1. (4 points) Let $\underline{\text{Set}}$ be the category of sets and all functions. Let $\mathscr{C}$ be a reasonably nice[1] category. Fix an object A in $\mathscr{C}$. Define a map

$$\begin{aligned} \text{Hom}_{\mathscr{C}}(A, -) : \mathscr{C} &\longrightarrow \underline{\text{Set}} \\ B &\longmapsto \text{Hom}_{\mathscr{C}}(A, B) \end{aligned}$$

We can extend this map to a map of morphisms

$$\begin{aligned} \mathscr{C} &\longrightarrow \underline{\text{Set}} \\ [f : B \to C] &\longmapsto [f_* : \text{Hom}_{\mathscr{C}}(A, B) \to \text{Hom}_{\mathscr{C}}(A, C)] \end{aligned}$$

to make it a covariant functor.[2] Explain how to define the map f_*, and verify that your construction is functorial.

[1] For set-theoretic reasons, we need $\mathscr{C}$ to be a *locally small category.* This holds for every category we will encounter.

[2] For each object A there is a **covariant** functor $\text{Hom}_{\mathscr{C}}(A, -)$ and a **contravariant** functor $\text{Hom}_{\mathscr{C}}(-, A)$. This question is about the first.