Name: ______________________________ Score (Out of 3 points):

1. Let X and Y be path-connected, locally path-connected, semi-locally simply-connected. Let $p_X : \tilde{X} \to X$ and $p_Y : \tilde{Y} \to Y$ be their universal covers.

   (a) (1 point) Explain why, for every continuous map $f : X \to Y$, there exists a continuous map $\tilde{f} : \tilde{X} \to \tilde{Y}$ that makes the following diagram commute.

$$\begin{array}{ccc} \tilde{X} & \xrightarrow{\tilde{f}} & \tilde{Y} \\ \big\downarrow{\scriptstyle p_X} & & \big\downarrow{\scriptstyle p_Y} \\ X & \xrightarrow{f} & Y \end{array}$$

   (b) (1 point) Is the map $\tilde{f}$ unique? Explain.

(c) (1 point) Consider the case that $X = S^1$ and $Y = S^1 \vee S^1$ as shown in Figure 1.

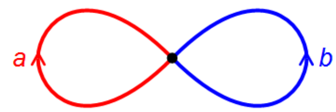

Figure 1: $Y = S^1 \vee S^1$

The universal cover of S^1 is $\mathbb{R}$, and the universal cover of $S^1 \vee S^1$ is shown in Figure 2.

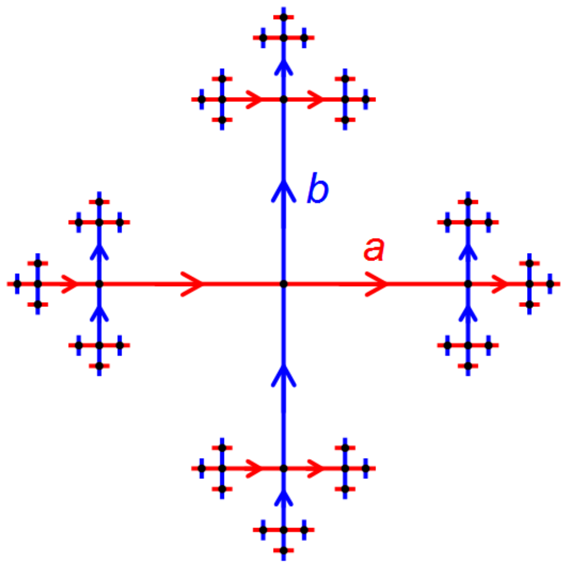

Figure 2: The universal cover $\tilde{Y}$ of $S^1 \vee S^1$

Let f be the constant-speed map that winds S^1 once (in the forward sense) around the loop a and then once (in the forward sense) around the loop b. Describe (informally) the corresponding map $\tilde{f}$ (or the set of all possible maps $\tilde{f}$) from $\mathbb{R}$ to the universal cover of $S^1 \vee S^1$.