Name: _____   Score (Out of 8 points):

1. A *terminal object* in a category $\mathscr{C}$ (if it exists) is an object T that satisfies the following universal property: for every object X in $\mathscr{C}$, there exists a unique morphism $f : X \to T$.

   (a) (3 points) Suppose that a terminal object T exists in a certain category $\mathscr{C}$. Prove that it is "unique up to unique isomorphism".

   (b) (2 points) Suppose T is a terminal object in a category $\mathscr{C}$. Prove that, for any object X and morphism $f : T \to X$, the morphism f is monic.

(c) (3 points) What are the terminal objects in the following categories? State a terminal object if one exists, otherwise write "Does not exist". No justification needed.

Let $\mathscr{C}$ be the category ...

$\underline{\text{Set}}$ of sets
and all functions:

$\underline{\text{Grp}}$ of groups
and group homomorphisms:

$\underline{\text{Fld}}$ of fields
and field homomorphisms:

$\underline{\text{Top}}$ of topological spaces
and continuous maps:

$\underline{\text{Top}}_*$ of based topological spaces
and based continuous maps:

$\underline{\text{hTop}}$ of topological spaces
and homotopy classes of continuous maps: